A pumping lemma for non-cooperative self-assembly.

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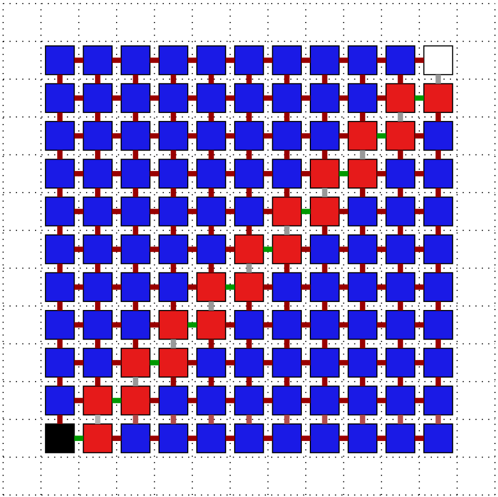
July 4, 2017

Definitions

- Consider a finite set $\mathcal{G}$ of *glue labels*.
- A temperature τ.
- A *tile type* t is an element $(\mathcal{G}, n)^4$ with $n \in \mathbb{N}$.
- A *tile set* T is a finite set of tile type.

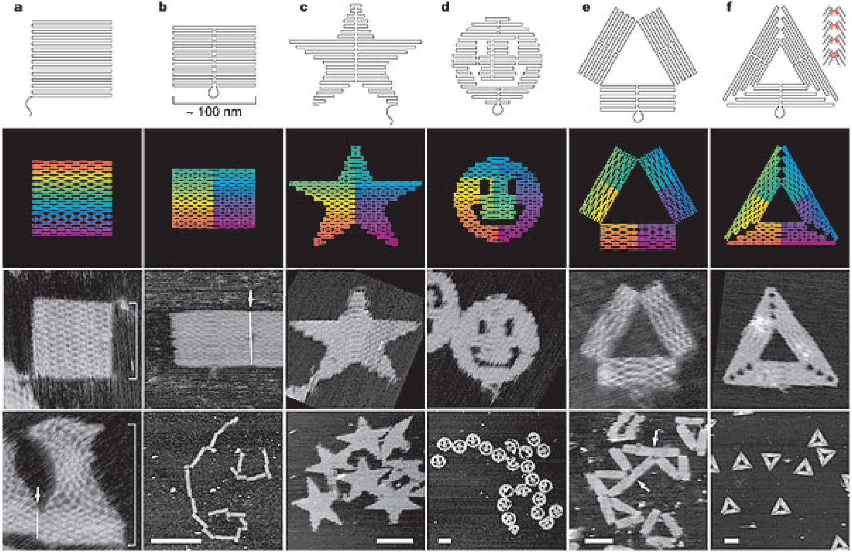
Definitions



At temperature 2,

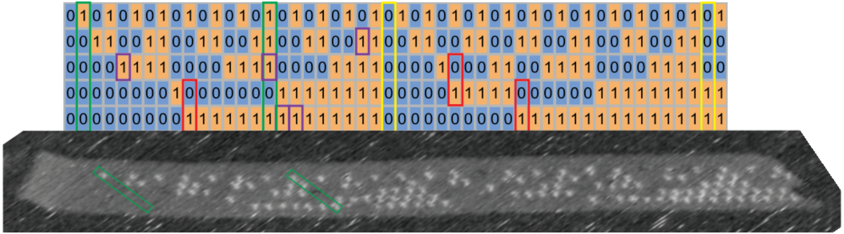
this tile set assembles all the squares.

Building biomolecular computers



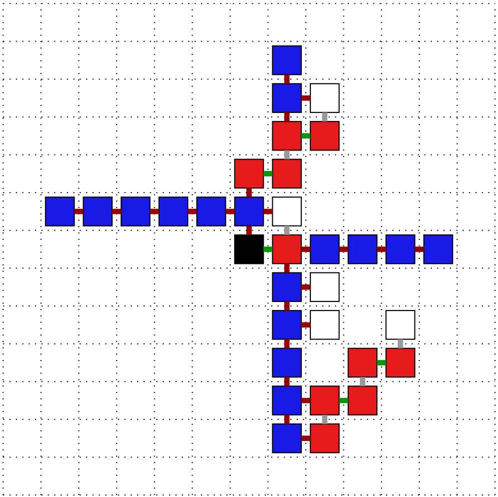
Paul W. K. Rothemund, Nature 2006

Programming DNA to build molecular computers

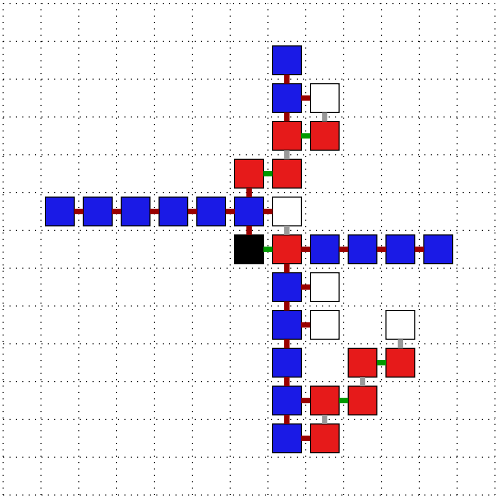


Constantine Evans, PhD. thesis, 2014

Temperature 1



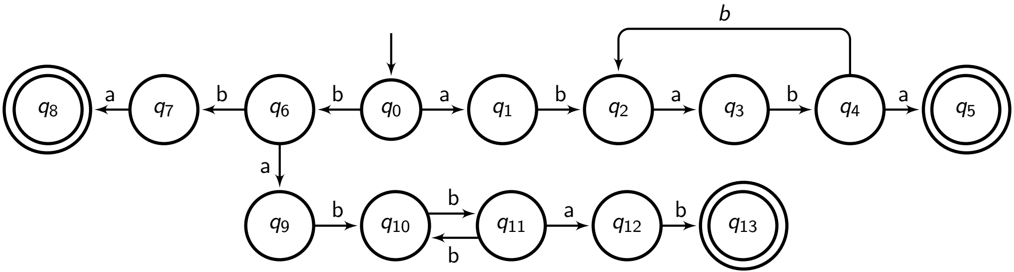
Temperature 1



Our objective: at temperature 1,

assembling a form without *hard coding* is *hard*.

Language Theory



This automata recognizes the language $\{a(bab)^*a + ba(bb)^*ab + bba\}$.

Pumping Lemma

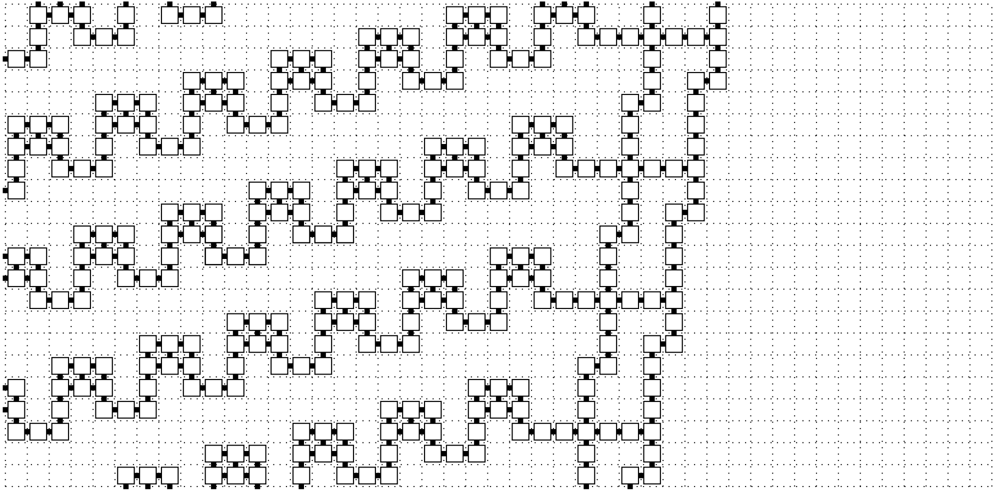
In language theory:

the pumping lemma implies than no computations are possible.

Our result:

a pumping lemma for tile assembly system at temperature 1.

Pumping lemma for temperature 1 (Objective)



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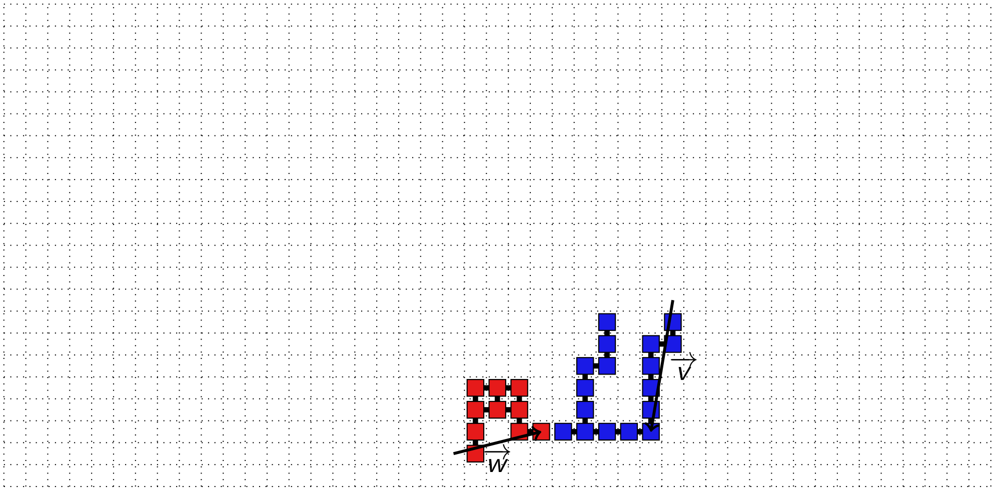

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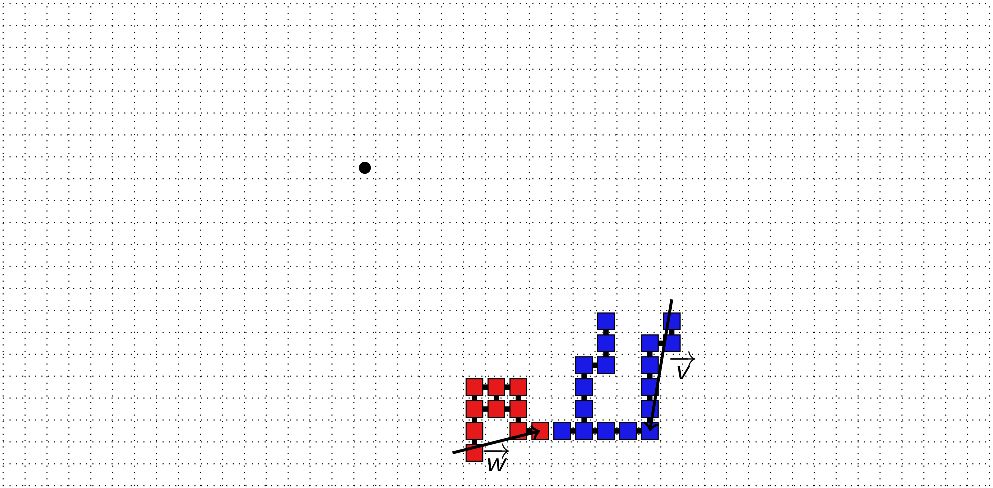

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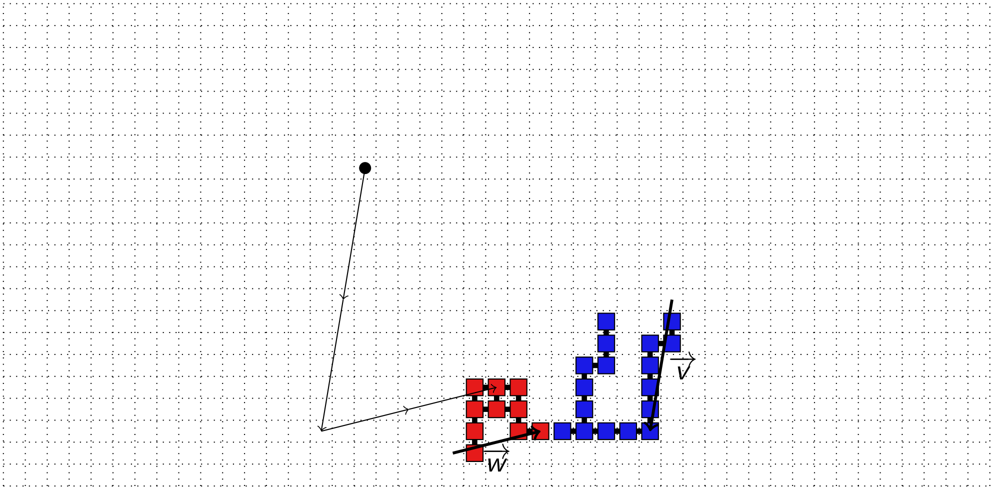

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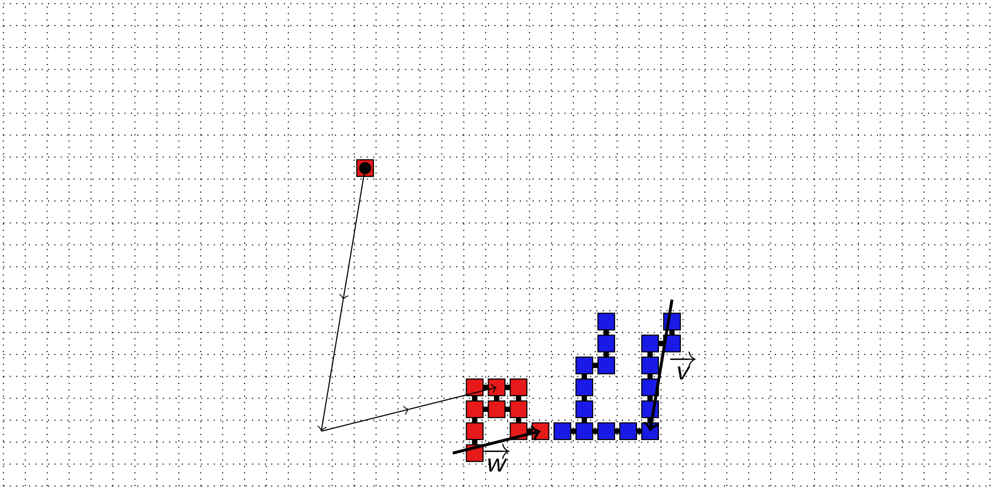
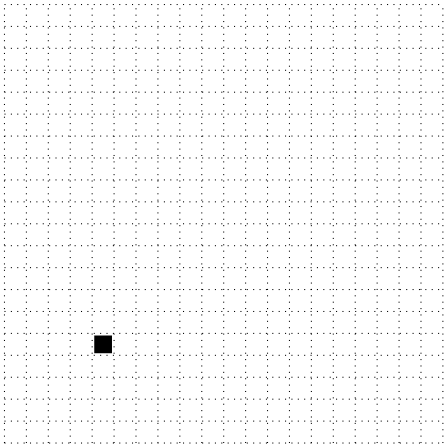


First obstacle: Branching



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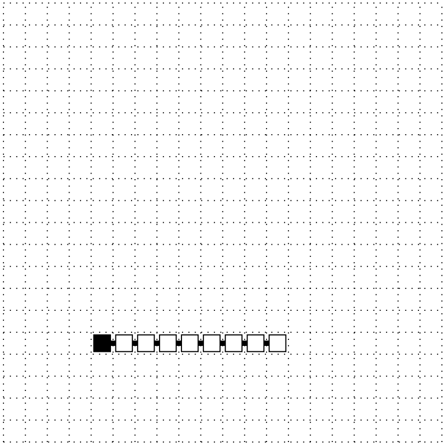

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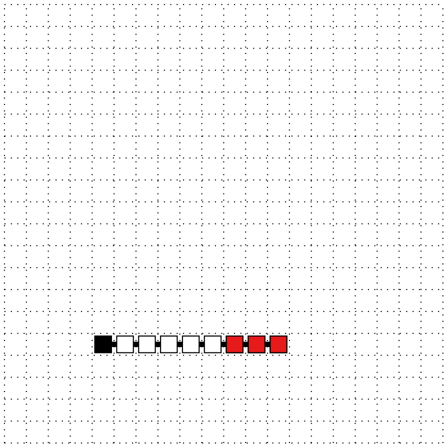

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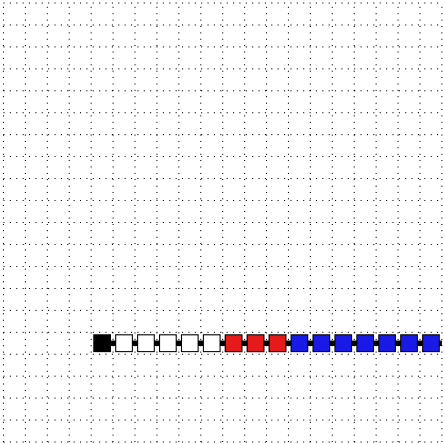

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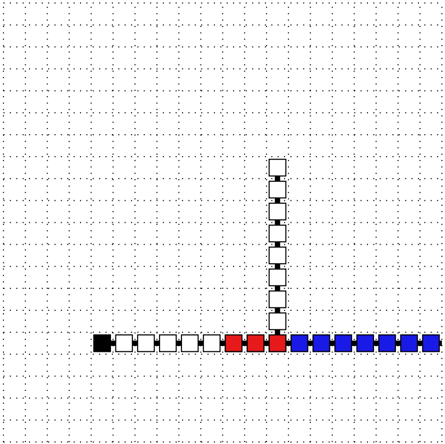

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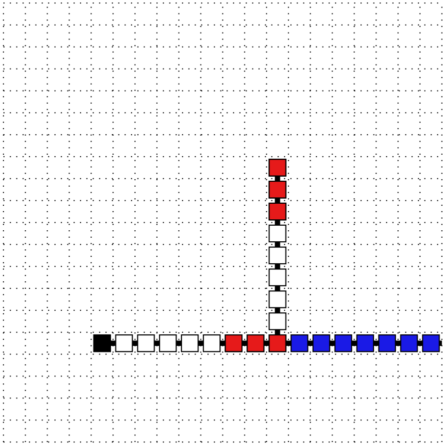

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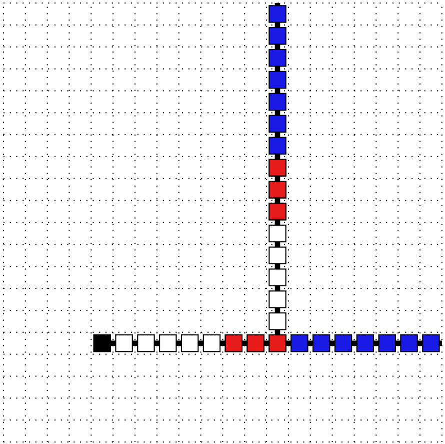

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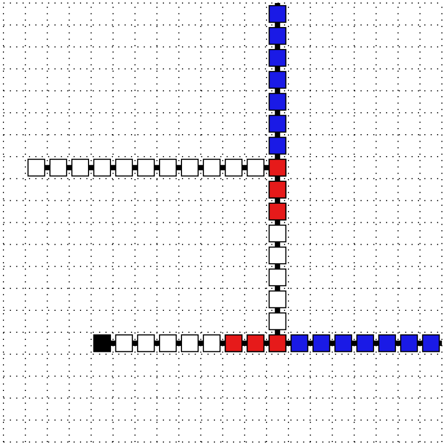

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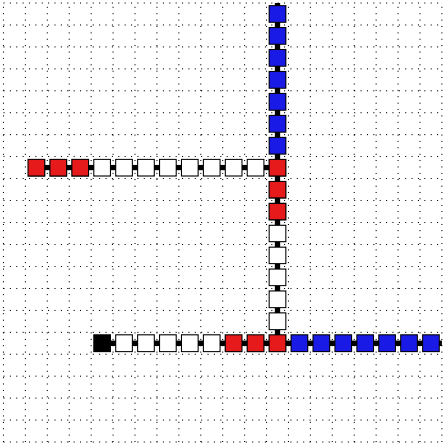

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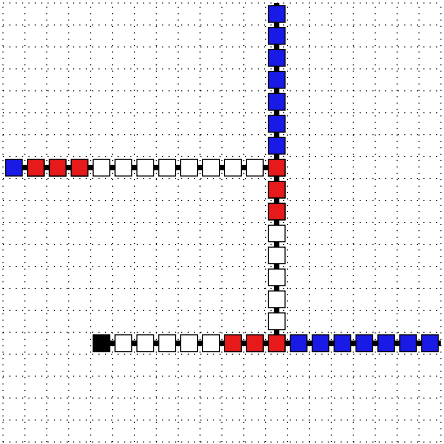

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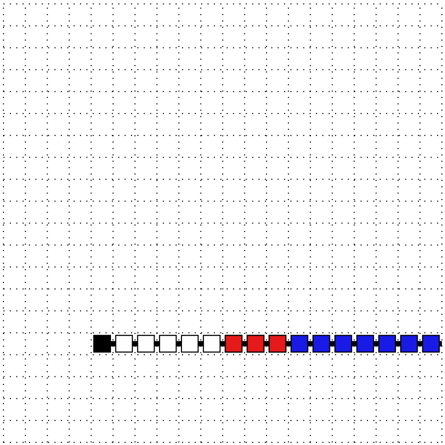

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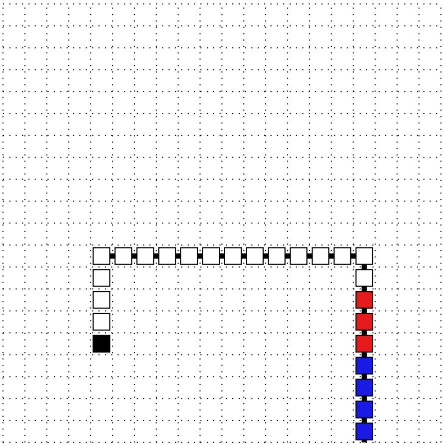

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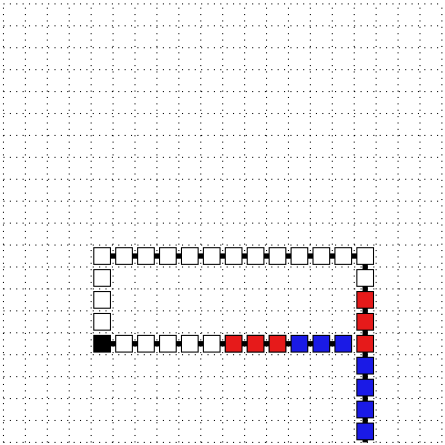

Deterministic case (unique terminal assembly)

By combining our result and Doty et al [TCS 2011].

Theorem

At temperature 1*, there exists only four kinds of terminal assembly of a deterministic tile assembly system.*

Non deterministic case

Theorem (Current result)

There exists a bound B depending on the number of tiles and the size of the seed such that at temperature 1, a path assembly growing at distance B of the seed is pumpable or fragile.

Non deterministic case

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There exists a bound B depending on the number of tiles and the size of the seed such that at temperature 1, a path assembly growing at distance B of the seed is pumpable or fragile.

Other results at temperature 1:

- a tile assembly system using polyominos can make computations (Fekete et al [SODA 2015]);
- a $3D$ tile assembly system can make computations (Cook et al [SODA 2011]);
- a probabilistic tile assembly system can make computations (Cook et al [SODA 2011]);
- an efficient path of length two times the number of tiles can be built (Meunier [UCNC 2015]).

Non deterministic case

Theorem (Objective)

At temperature 1*, there exists a simple terminal assembly.*

To obtain this result we have to deal with fragility (see Meunier and Woods [STOC 2017]) and branching.