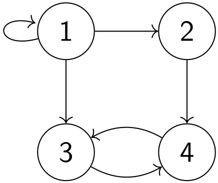
On the Cost of Sequentialisation of Automata Networks

Florian Bridoux (LIF Marseille France)

Automata networks (ANs)



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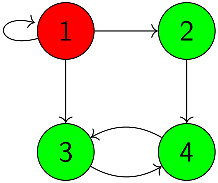

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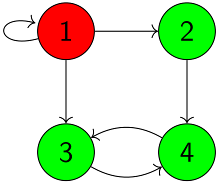

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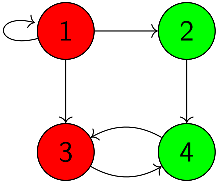

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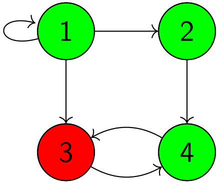

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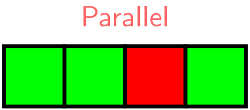

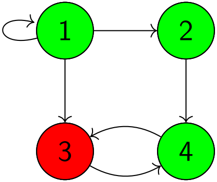

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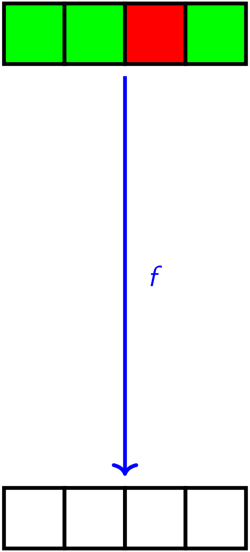

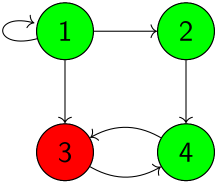

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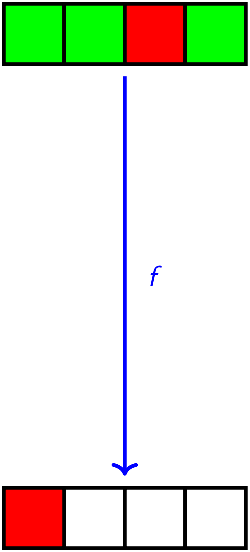

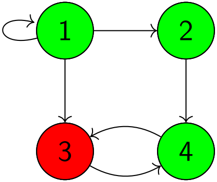

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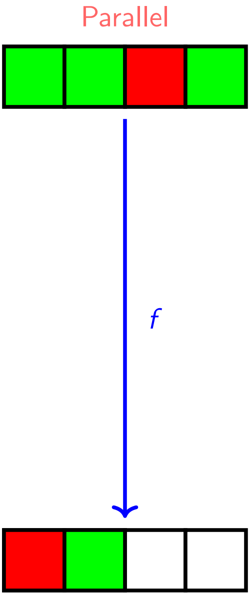

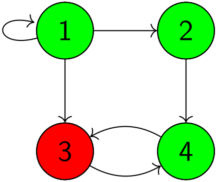

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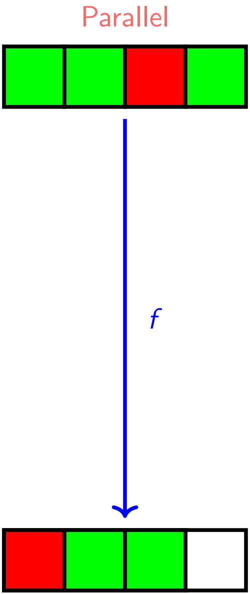

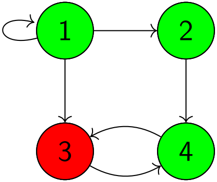

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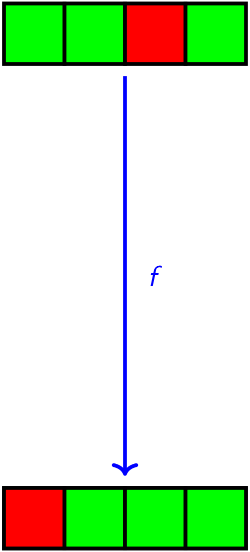

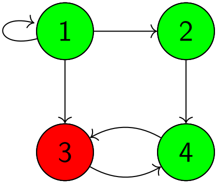

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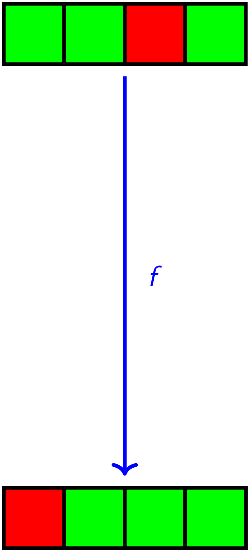

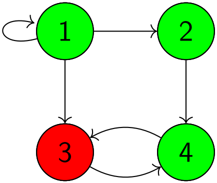

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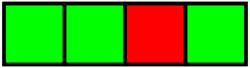

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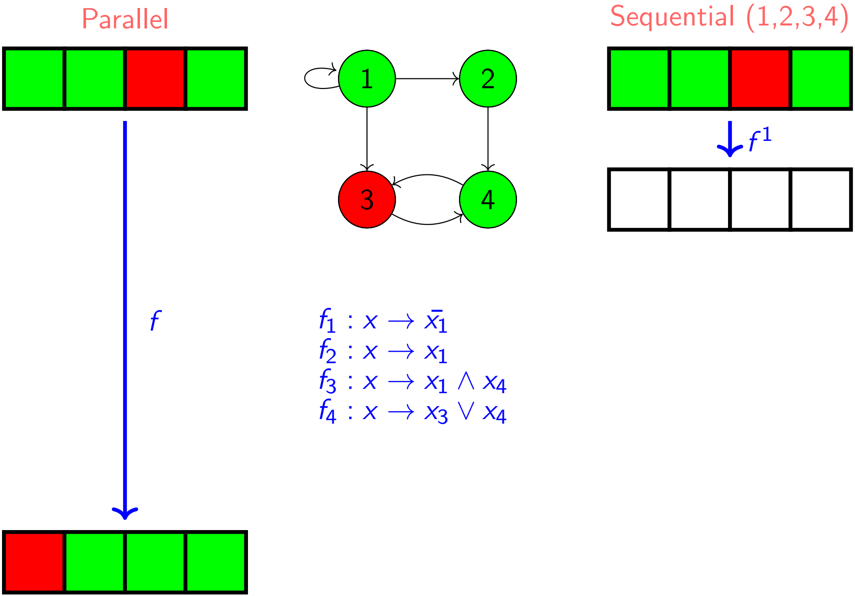

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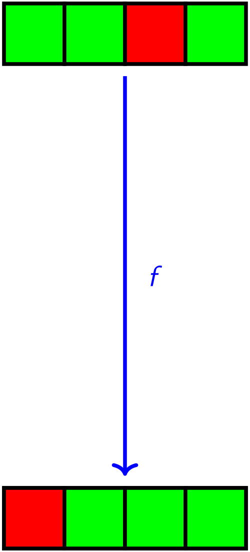

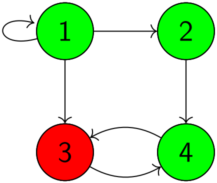

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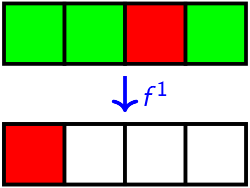

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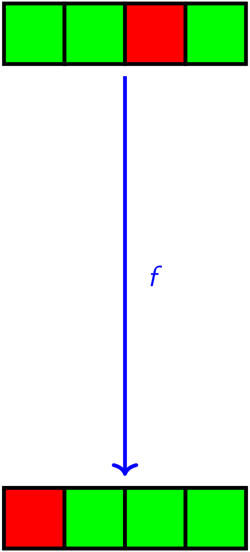

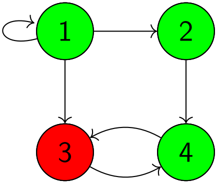

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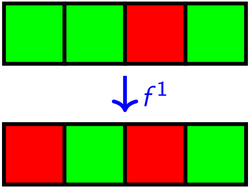

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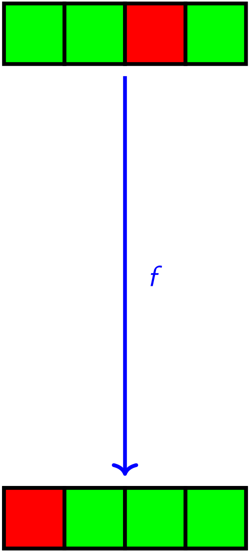

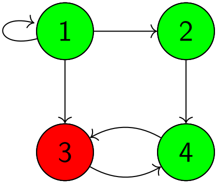

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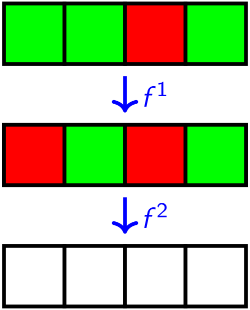

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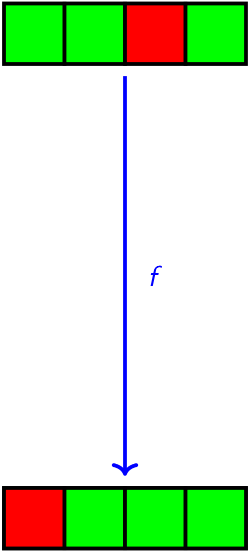

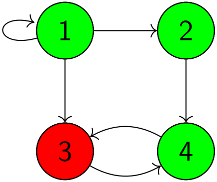

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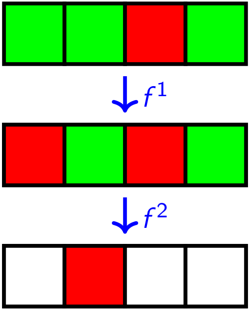

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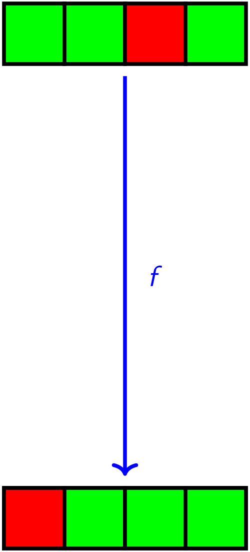

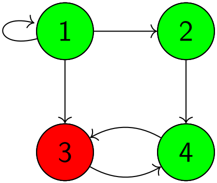

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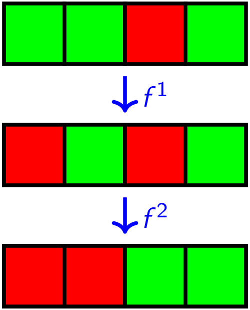

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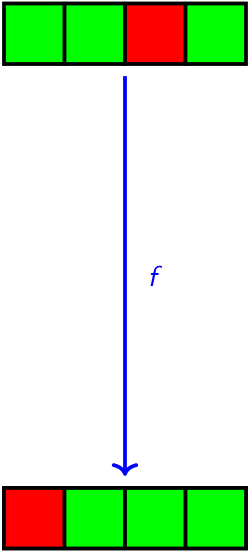

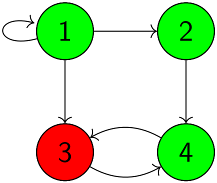

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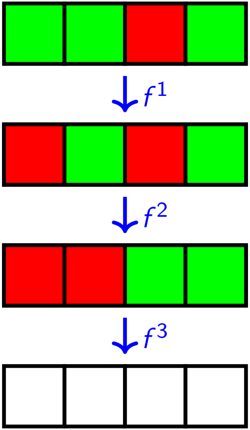

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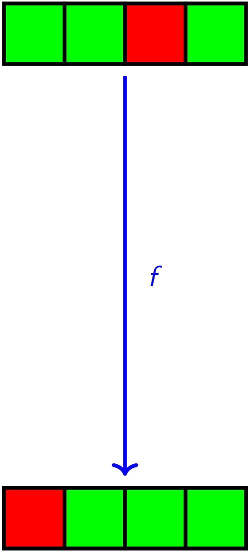

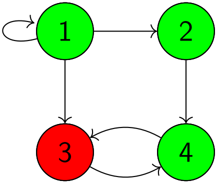

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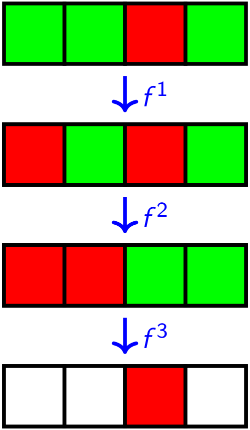

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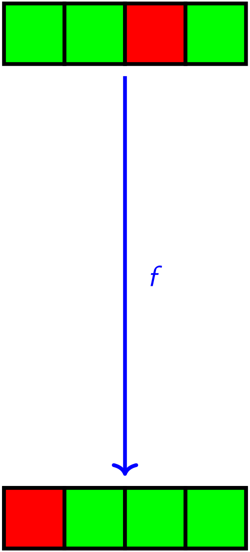

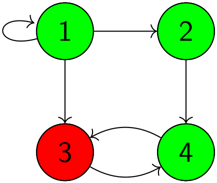

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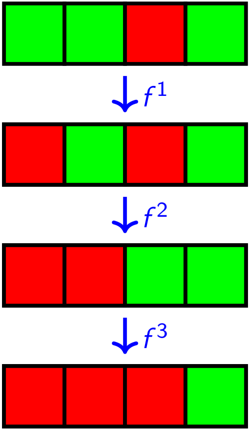

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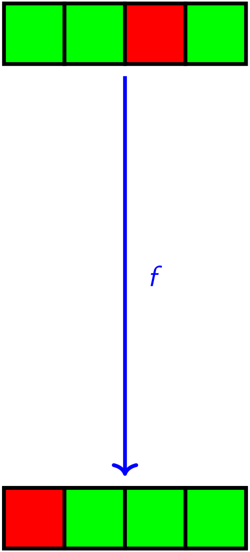

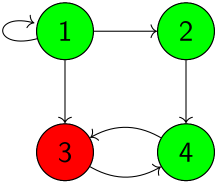

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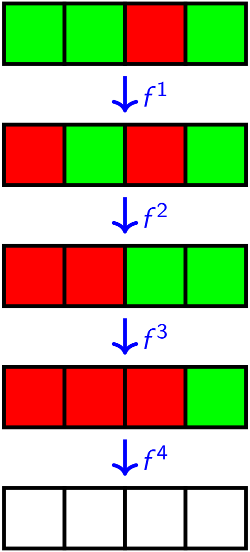

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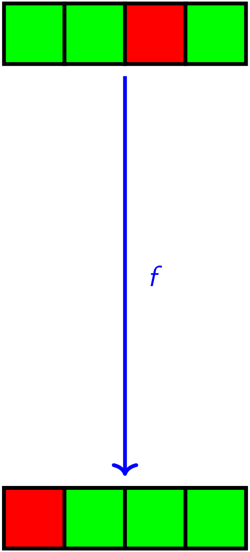

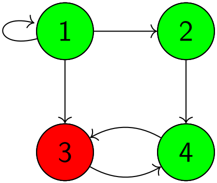

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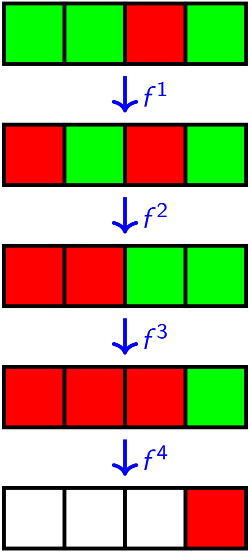

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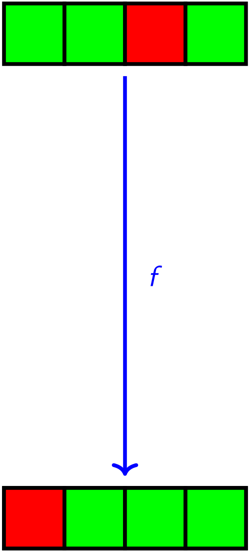

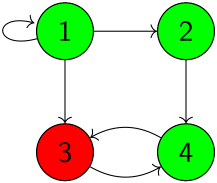

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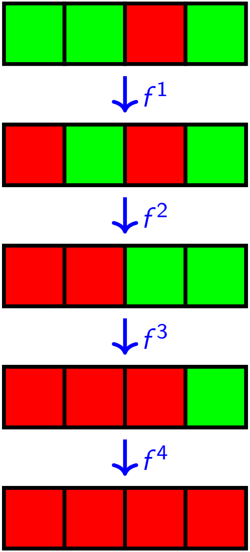

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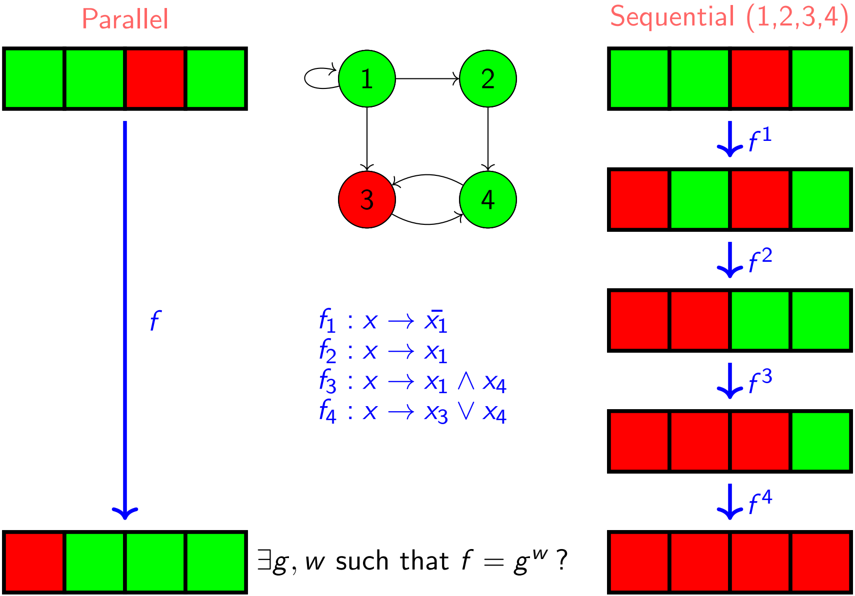
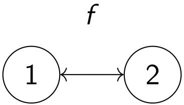


The cost of sequentialisation



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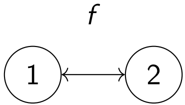

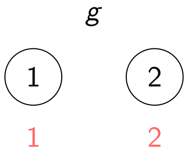

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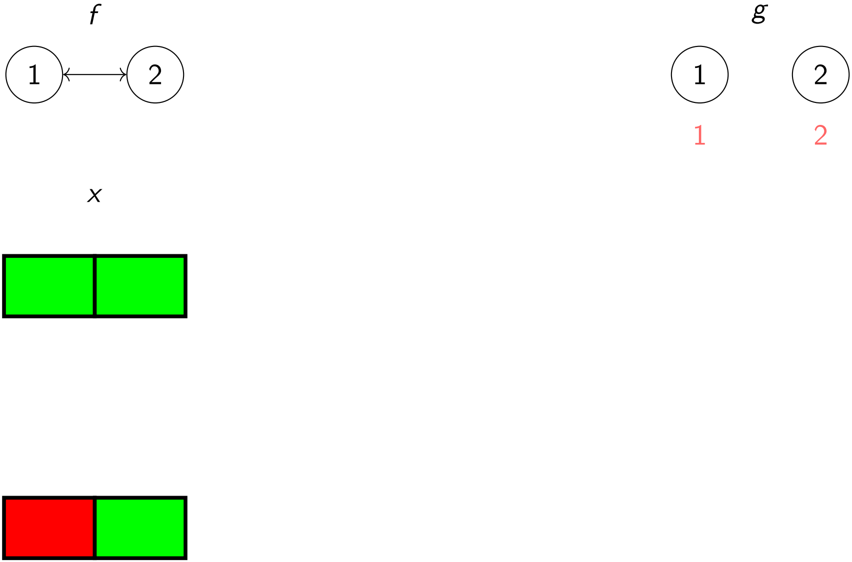

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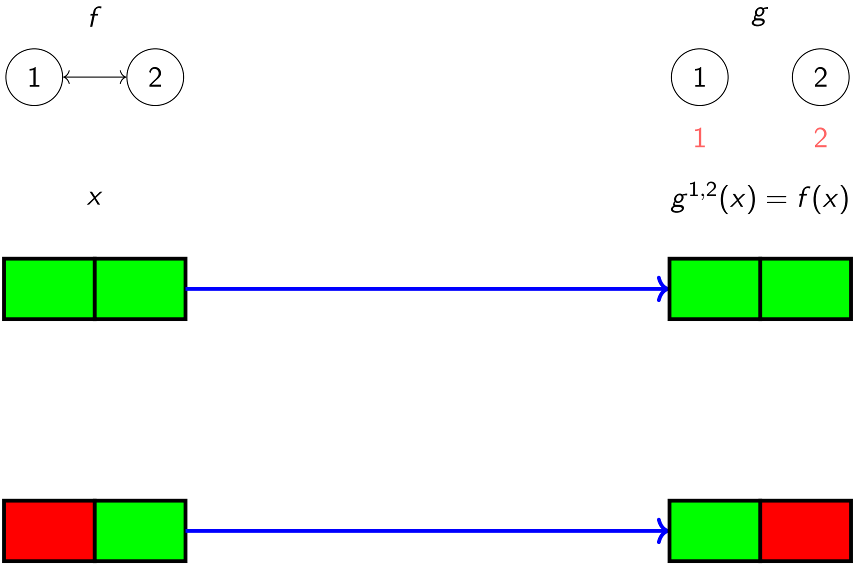
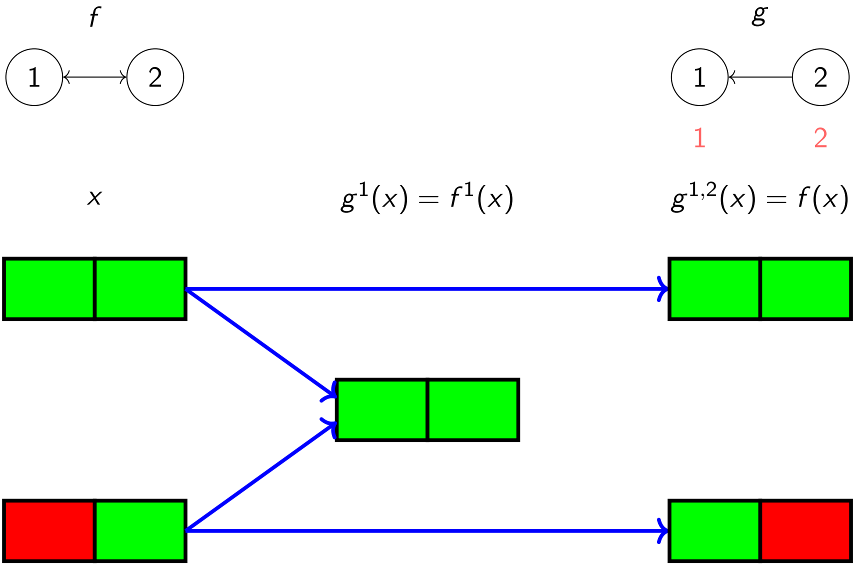


The cost of sequentialisation



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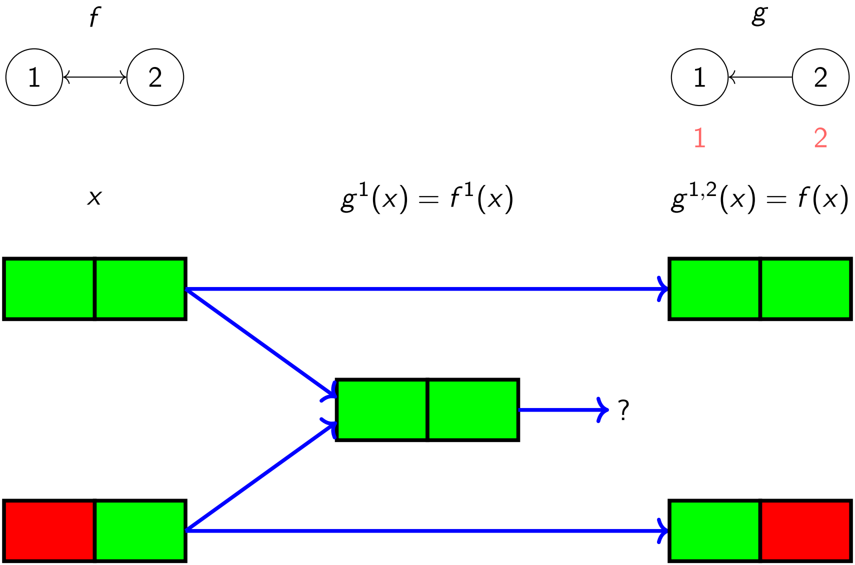

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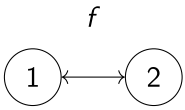

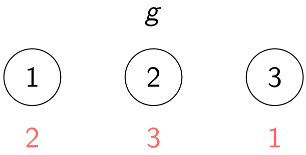

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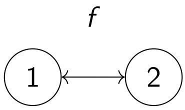

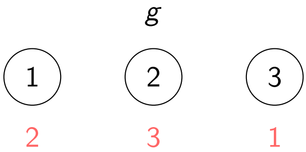

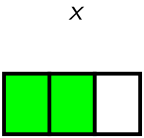

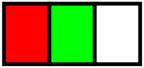

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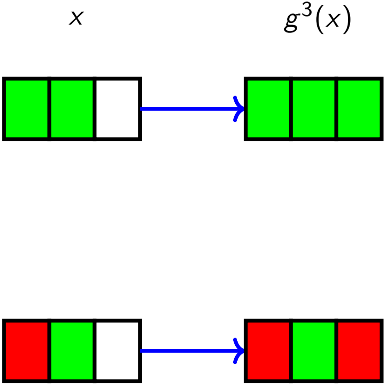

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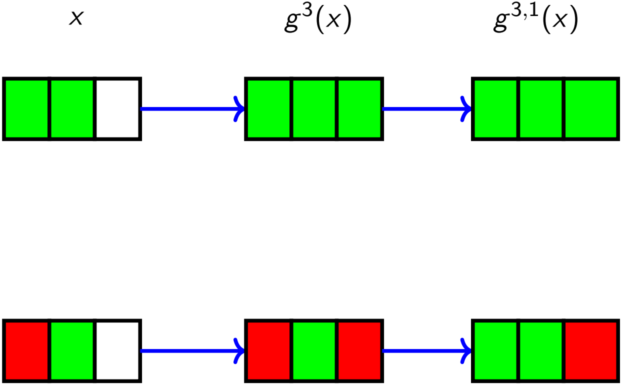

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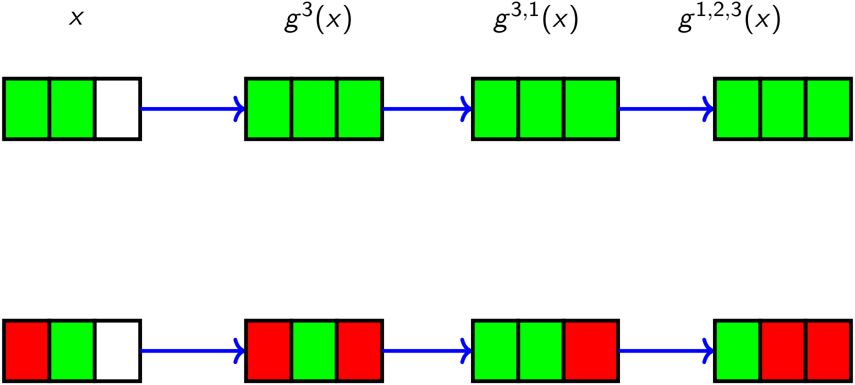

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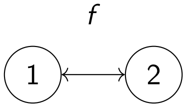
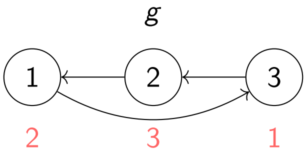
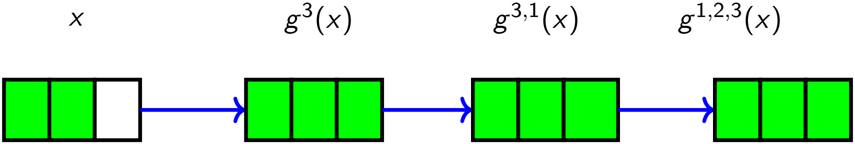
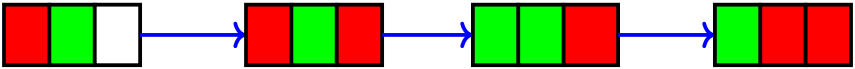






$$f \circ pr_{[n]} = pr_{[n]} \circ g^w$$



The cost of sequentialisation

$\kappa(f, w)$: number of additional nodes required to sequentialisate f with the schedule w.

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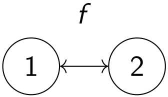
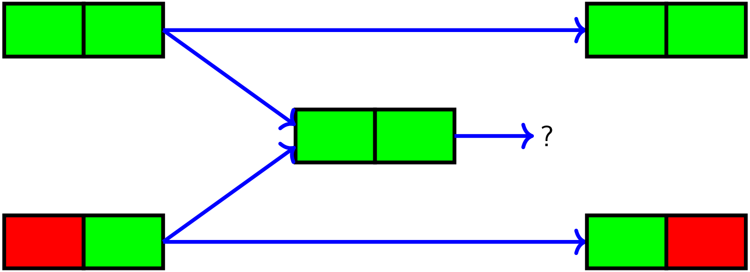
$\kappa^{\min}_{|A|,n} = \max(\{\kappa^{\min}(f, w) \mid f : A^n \to A^n\})$

Confusion graph $G_{f,w}$





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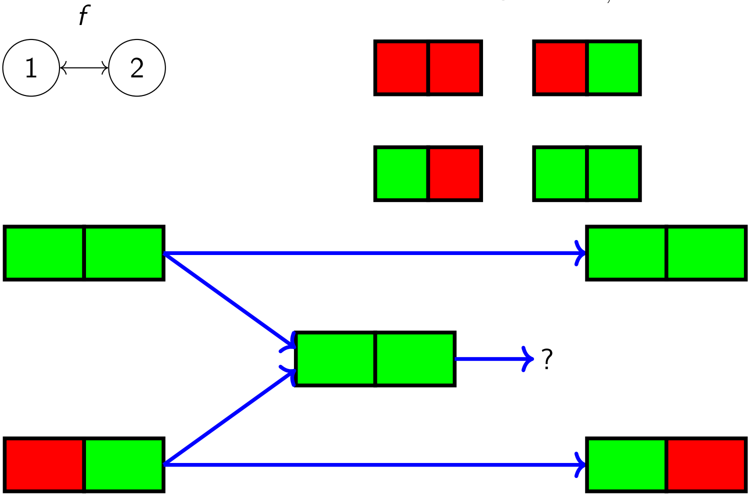

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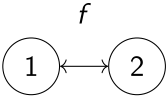

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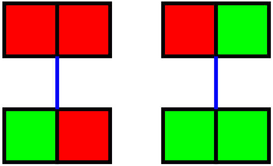

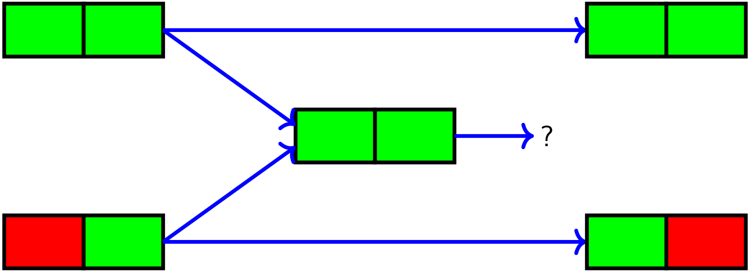

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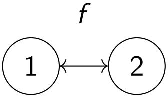

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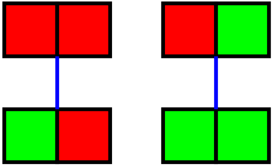
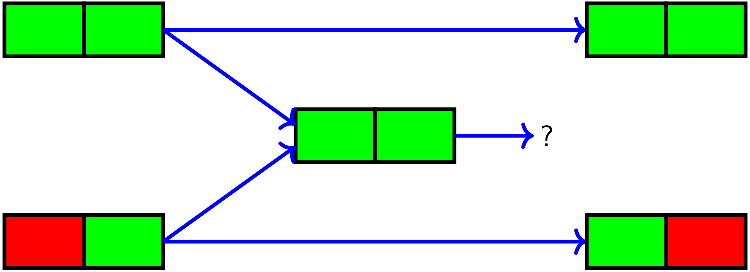
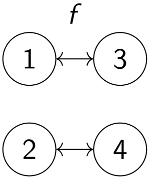


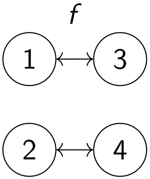


Th. $\kappa(f, w) = \lceil \log_{|A|}(\chi(G_{f,w})) \rceil$



$w = (1, 2, 3, 4)$

κ_n : Lower bound and conjecture



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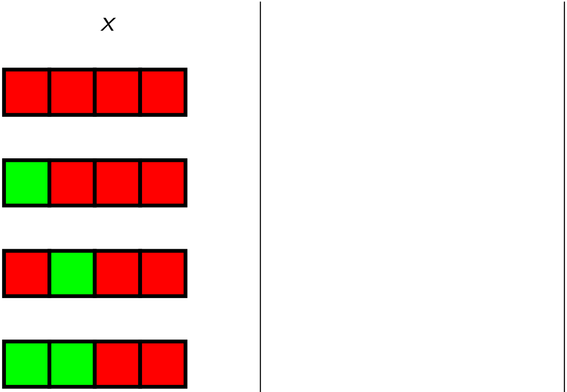

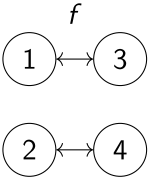

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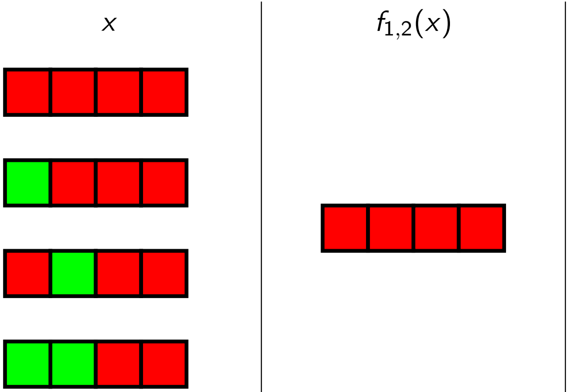

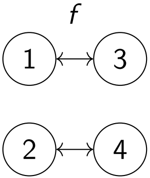

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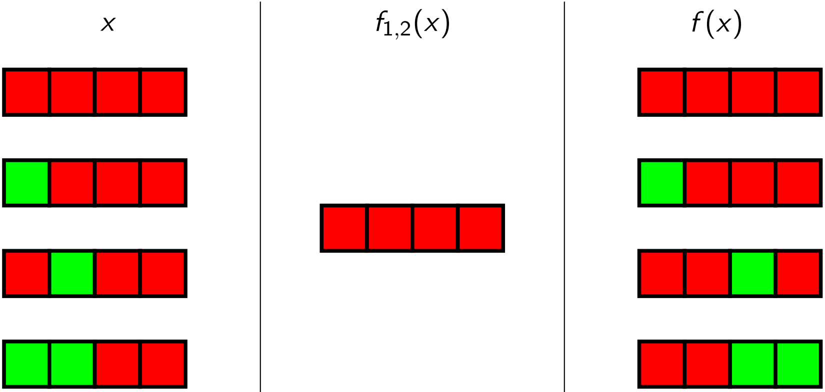

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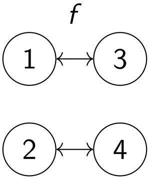

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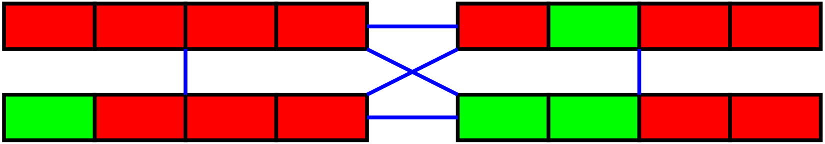

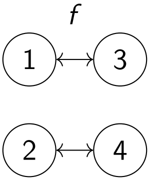

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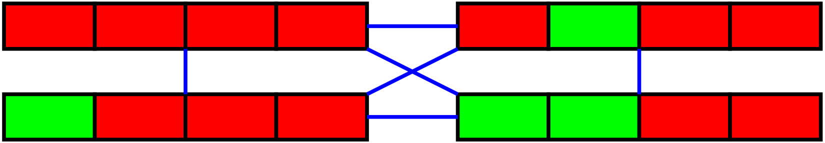

$\omega(G_{f,w}) \geq 2^{n/2} = 2^2$

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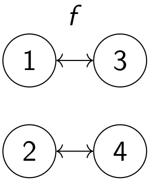

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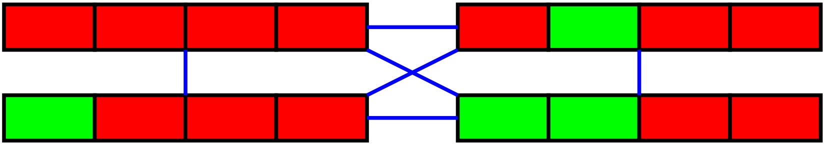

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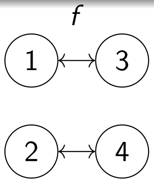

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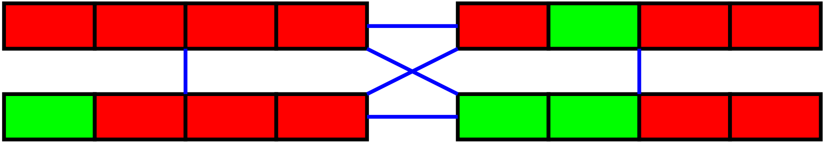

$\omega(G_{f,w}) \geq 2^{n/2} = 2^2$

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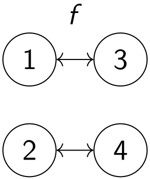

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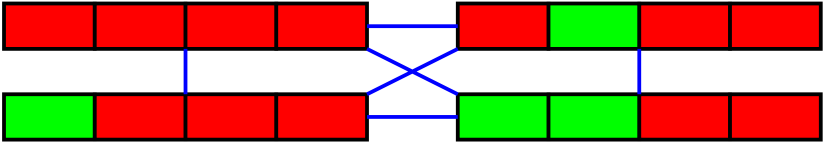
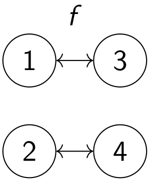


Theorem. $\lfloor n/2 \rfloor \leq \kappa_n$



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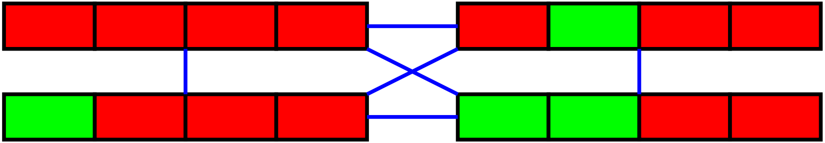


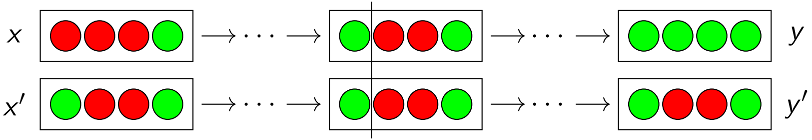
Theorem. $\lfloor n/2 \rfloor \leq \kappa_n$

Conjecture. $\kappa_n = \lfloor n/2 \rfloor$

Upper bound for κ_q, n : $n/2 + \log(n)$

Let $E^1, \ldots, E^p$ be a partition of A^n and $x^1 \in E^1, \ldots, x^p \in E^p$ such that :

- if $f(x) = f(x^i)$ and $x_{[n/2,n]} = x^1_{[n/2,n]}$ then $x \in E^i$.



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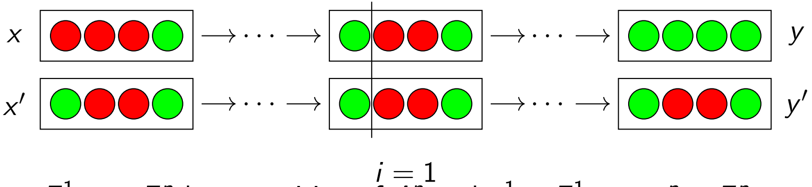

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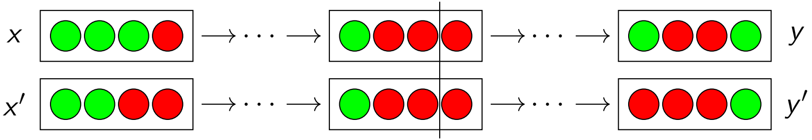
$d_{[1,n/2]}(E^1) \leq q^{n/2}$.



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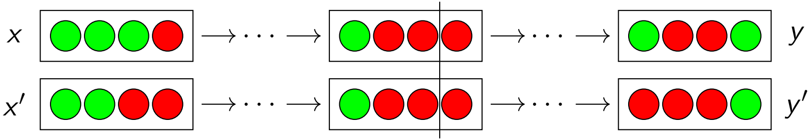

$i = 3$

Let $E^1, \ldots, E^p$ be a partition of A^n and $x^1 \in E^1, \ldots, x^p \in E^p$ such that :

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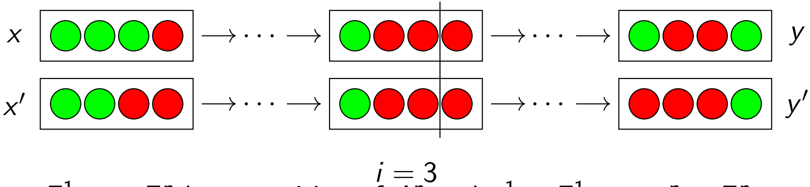
$$\forall i \in [n/2, n], d_i(E^1) \leq q^{n-i} * q^{i-n/2} = q^{n/2}.$$



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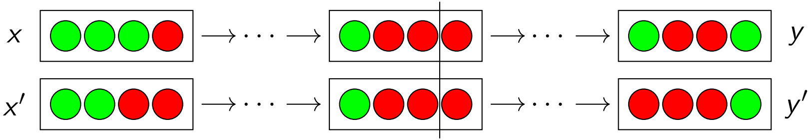
$$d(E^1) \leq (n/2+1)q^{n/2}.$$

Upper bound for κ_q, n : $n/2 + \log(n)$



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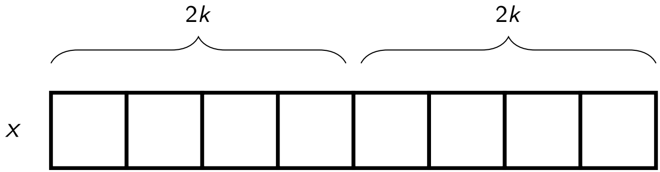
$d(E^1) \leq (n/2 + 1)q^{n/2}$.

Theorem. $\kappa_{q,n} \leq n/2 + \log_q(n/2 + 1)$.

Lower bound for $\kappa_{2,n}^{min}$: $n/4$



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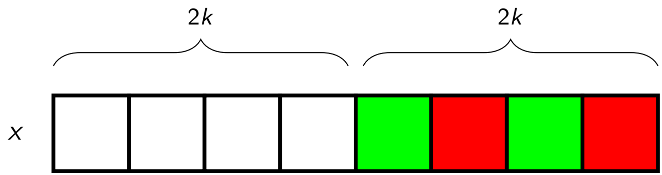
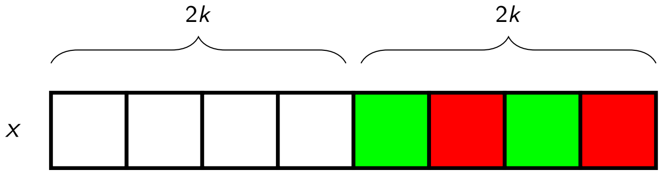


- $\forall i \in [n, 2], f_i(x) = x_i$

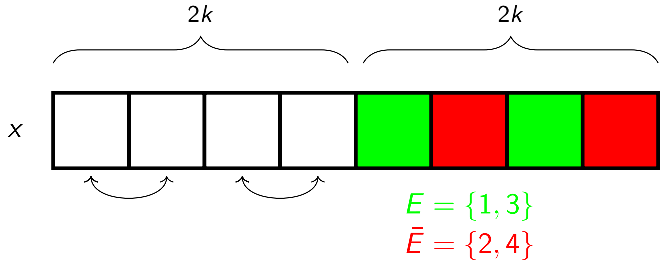


$E = \{1, 3\}$

$\bar{E} = \{2, 4\}$

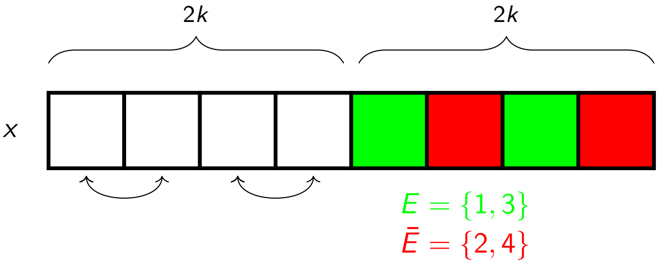
- $\forall i \in [n, 2], f_i(x) = x_i$

Lower bound for $\kappa_{2,n}^{min}$: $n/4$



- $\forall i \in [n, 2], f_i(x) = x_i$
- if $|E| = |\bar{E}| = k$:
  - $f_E(x) = x_{\bar{E}}$
  - $f_{\bar{E}}(x) = x_E$

Lower bound for $\kappa_{2,n}^{min}$: $n/4$



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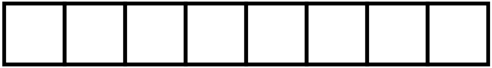
- if $|E| = |\bar{E}| = k$:
  - $f_E(x) = x_{\bar{E}}$
  - $f_{\bar{E}}(x) = x_E$
- else :
  - $f(x) = x$

Lower bound for $\kappa^{min}_{2,n}$: $n/4$

$$w = (4, 2, 1, 3, 5, 6, 7, 8)$$

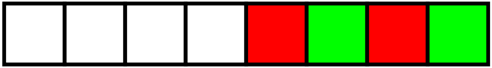
x



Lower bound for $\kappa_{2,n}^{min}$: $n/4$

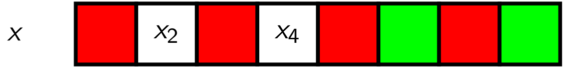
$w = (4, 2, 1, 3, 5, 6, 7, 8)$

x



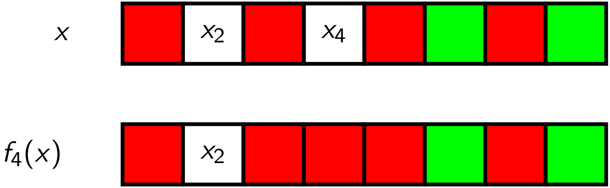
Lower bound for $\kappa_{2,n}^{min}$: $n/4$

$w = (4, 2, 1, 3, 5, 6, 7, 8)$



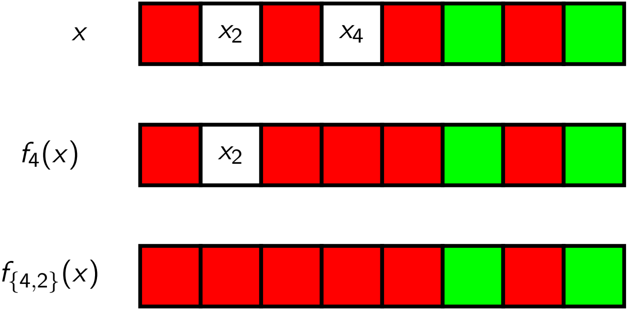
Lower bound for $\kappa_{2,n}^{min}$: $n/4$

$w = (4, 2, 1, 3, 5, 6, 7, 8)$



Lower bound for $\kappa_{2,n}^{min}$: $n/4$

$$w = (4, 2, 1, 3, 5, 6, 7, 8)$$



Lower bound for $\kappa_{2,n}^{min}$: $n/4$

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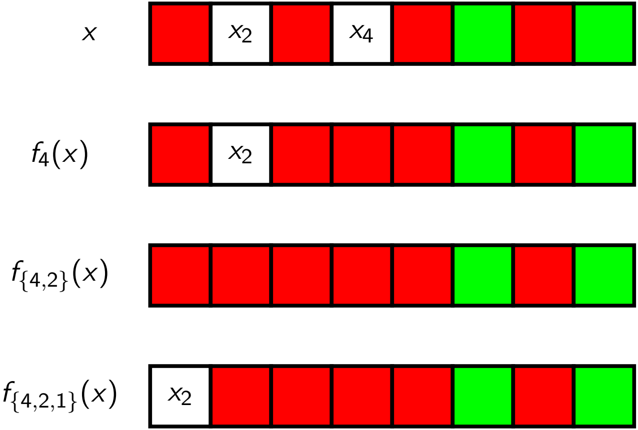
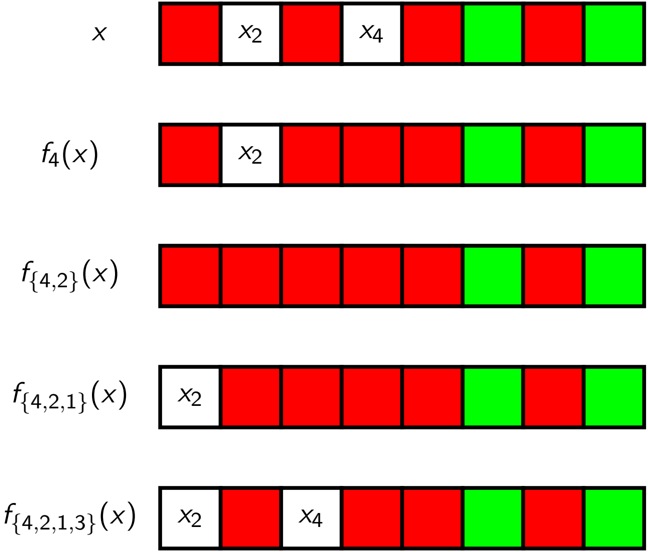
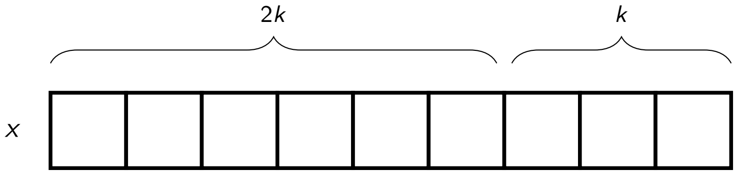


Lower bound for $\kappa_{2,n}^{min}$: $n/4$

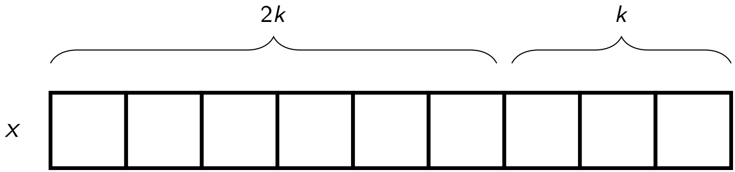
$w = (4, 2, 1, 3, 5, 6, 7, 8)$



Lower bound for $\kappa_{2,n}^{min}$: $n/3$

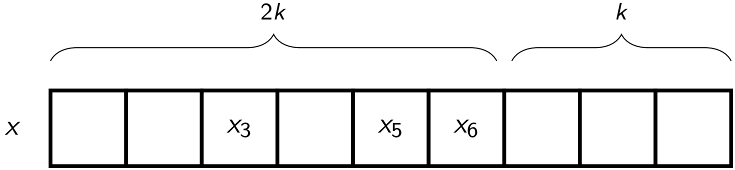


Lower bound for $\kappa^{min}_{2,n}$: $n/3$



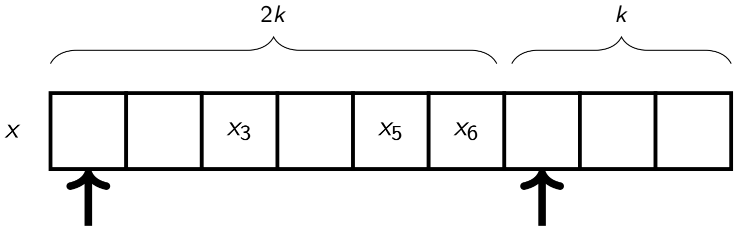
$E = \{3, 5, 6\}$
$\bar{E} = \{1, 2, 4\}$

Lower bound for $\kappa_{2,n}^{min}$: $n/3$



$$E = \{3, 5, 6\}$$
$$\bar{E} = \{1, 2, 4\}$$

Lower bound for $\kappa_{2,n}^{min}$: $n/3$



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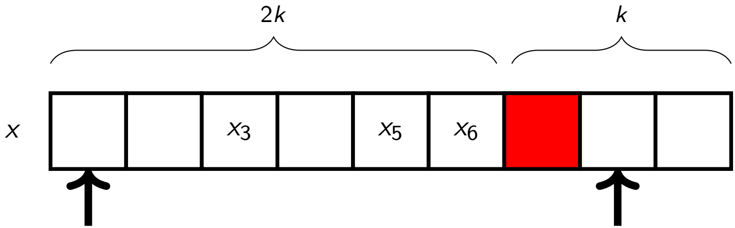

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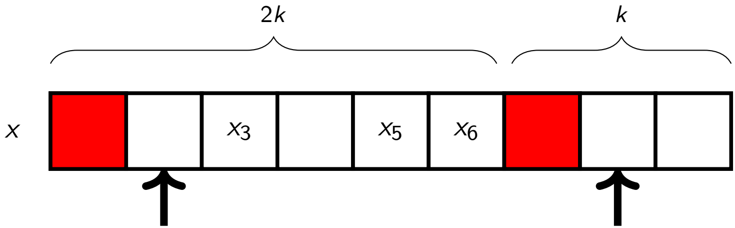
Lower bound for $\kappa_{2,n}^{min}$: $n/3$



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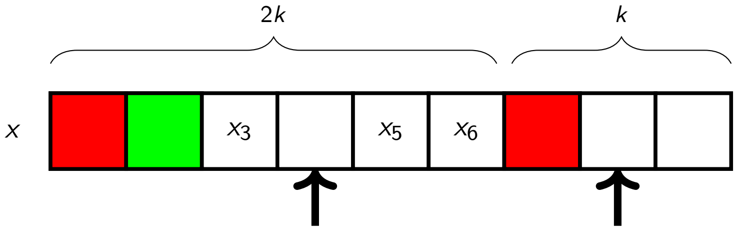

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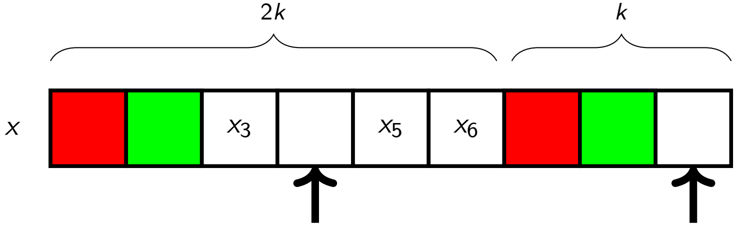

$E = \{3, 5, 6\}$

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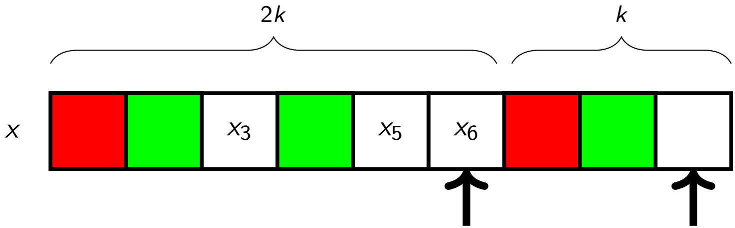

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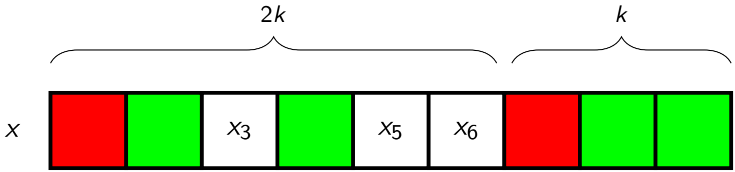
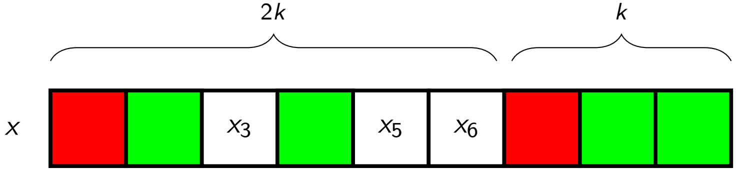
Lower bound for $\kappa_{2,n}^{min}$: $n/3$



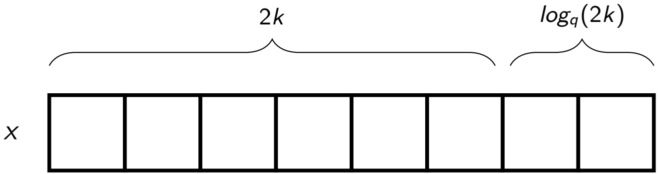
$E = \{3, 5, 6\}$
$\bar{E} = \{1, 2, 4\}$

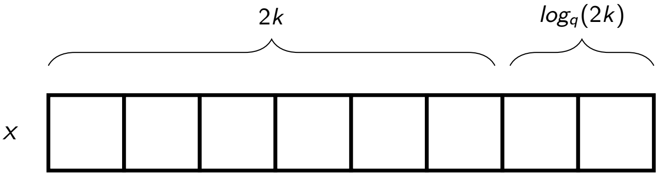


$$E = \{3, 5, 6\}$$
$$\bar{E} = \{1, 2, 4\}$$

Theorem. $\kappa_{2,n}^{\min} \geq n/3$

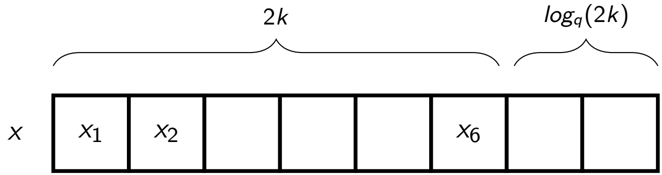
Lower bound for $\kappa_{q,n}^{\min}$ *when* $q \geq 4 : n/2 - \log_q(n)$





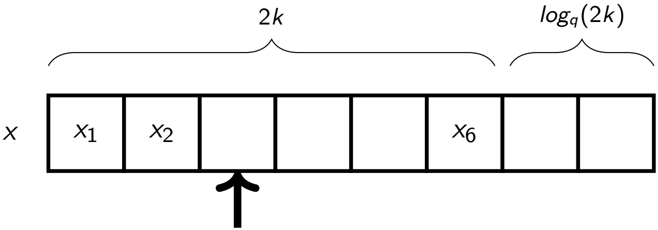
$E = \{1, 2, 6\}$
$\bar{E} = \{3, 4, 5\}$

Lower bound for $\kappa^{\min}_{q,n}$ *when* $q \geq 4 : n/2 - \log_q(n)$



$E = \{1, 2, 6\}$
$\bar{E} = \{3, 4, 5\}$

Lower bound for $\kappa_{q,n}^{\min}$ *when* $q \geq 4 : n/2 - \log_q(n)$



$E = \{1, 2, 6\}$
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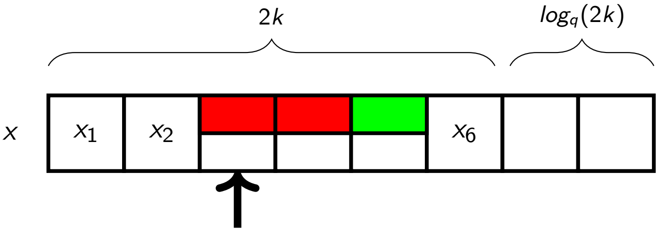
Lower bound for $\kappa^{\min}_{q,n}$ *when* $q \geq 4 : n/2 - \log_q(n)$



$E = \{1, 2, 6\}$
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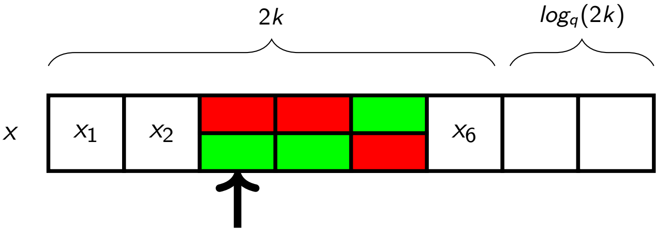
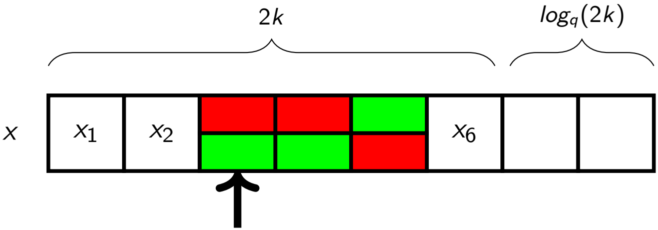
Lower bound for $\kappa^{\min}_{q,n}$ *when* $q \geq 4 : n/2 - \log_q(n)$



$E = \{1, 2, 6\}$
$\bar{E} = \{3, 4, 5\}$

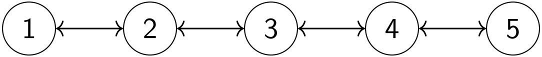
Lower bound for $\kappa^{\min}_{q,n}$ when $q \geq 4 : n/2 - \log_q(n)$



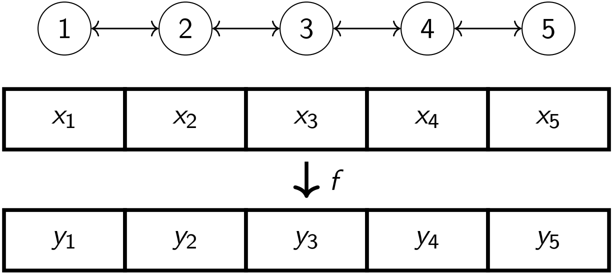
$E = \{1, 2, 6\}$
$\bar{E} = \{3, 4, 5\}$

Theorem. When $q \geq 4$, $\kappa^{\min}_{q,n} \geq n/2 - \log_q(n)$

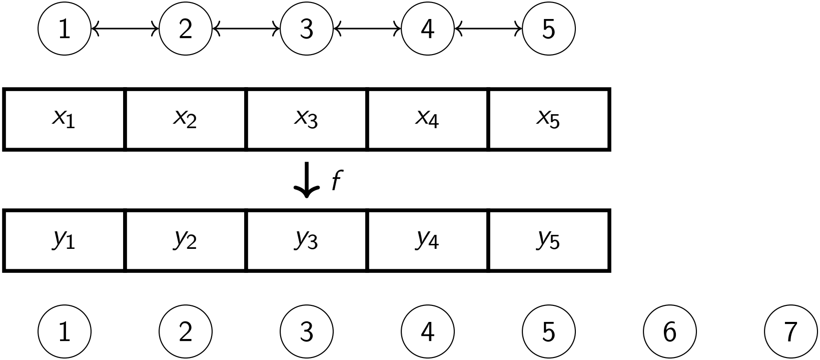
$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$

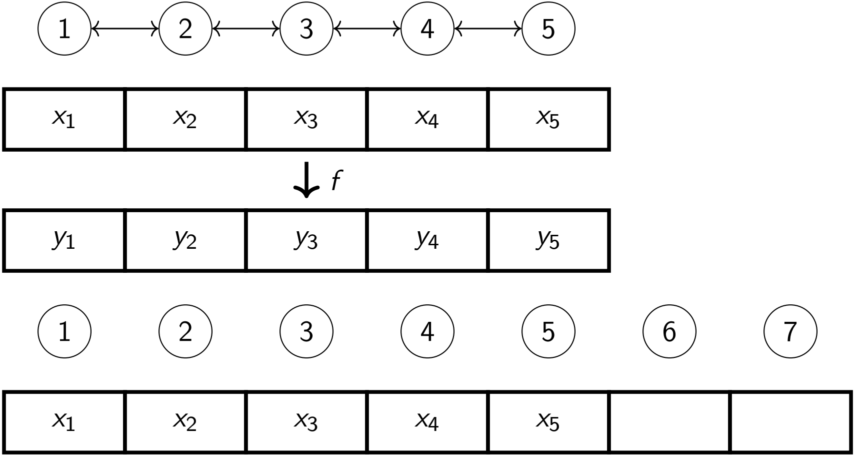


$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$



$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$




1
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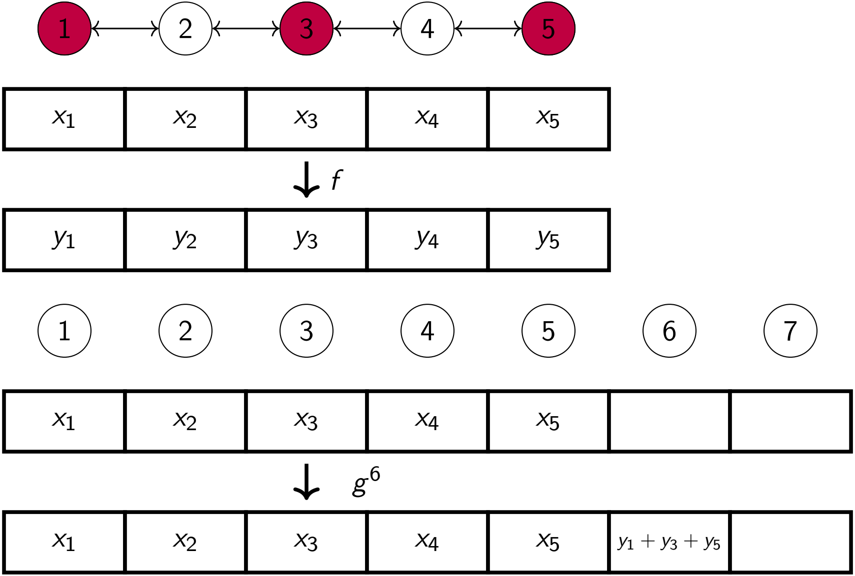
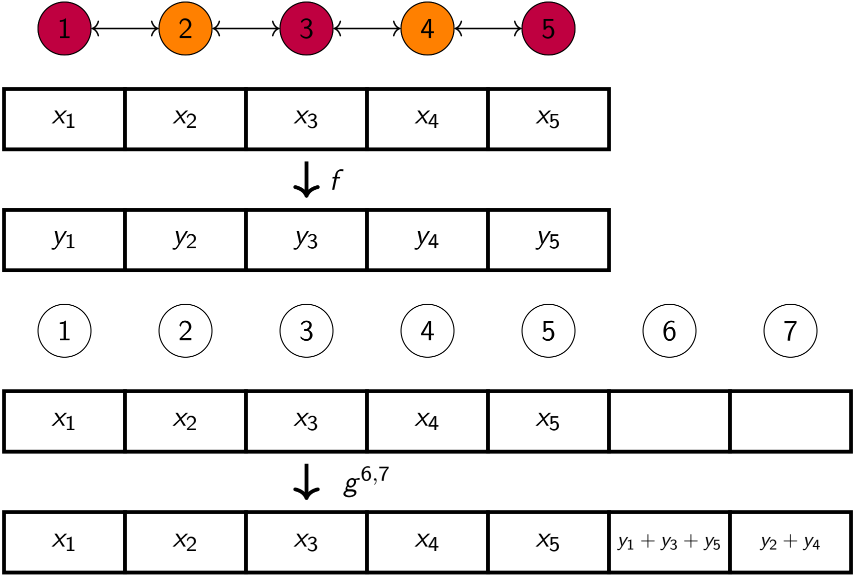
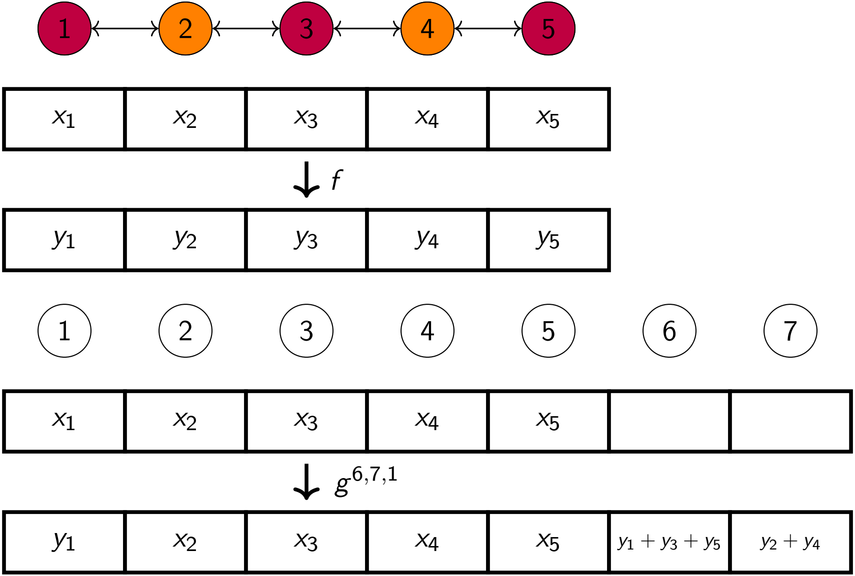
$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$



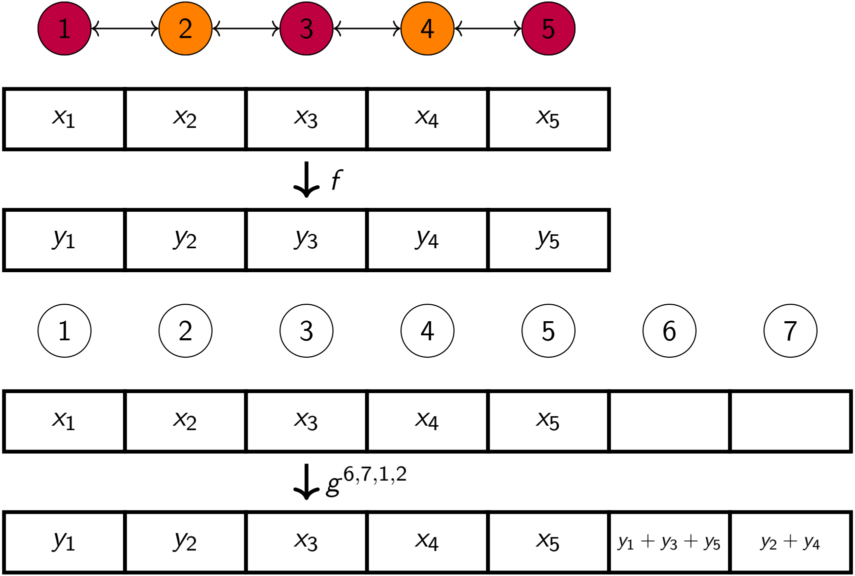
$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$



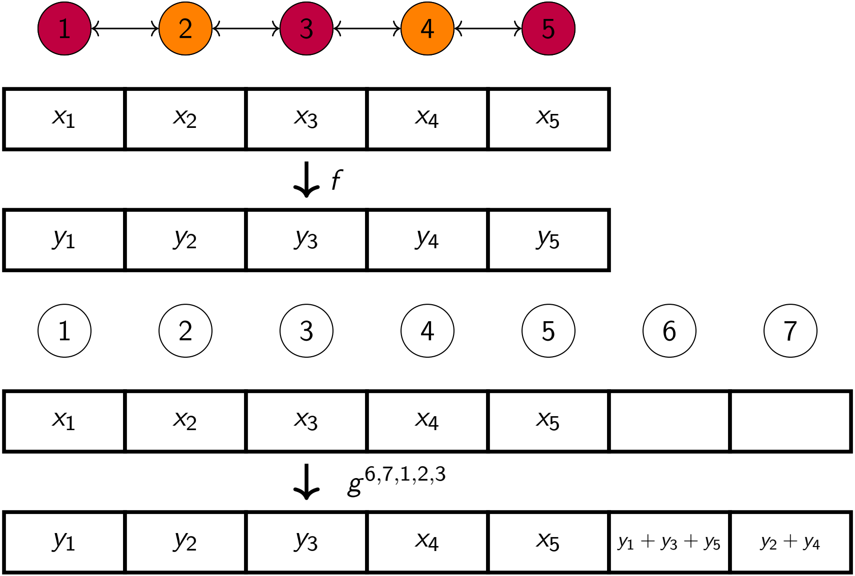
$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$



$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$



$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$



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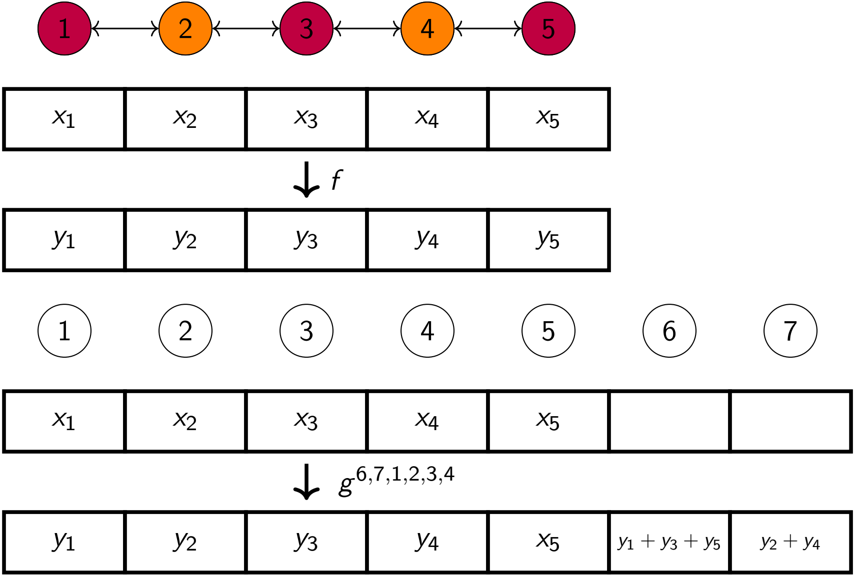
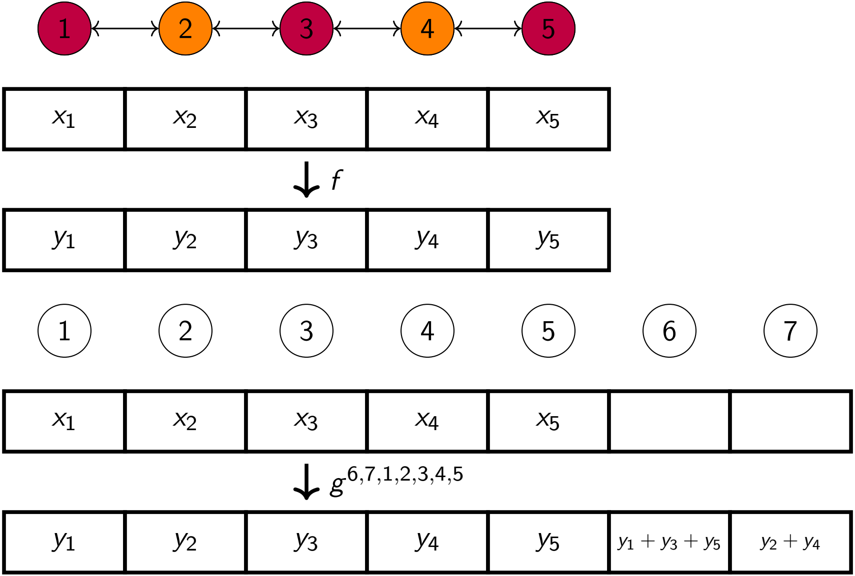


$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$



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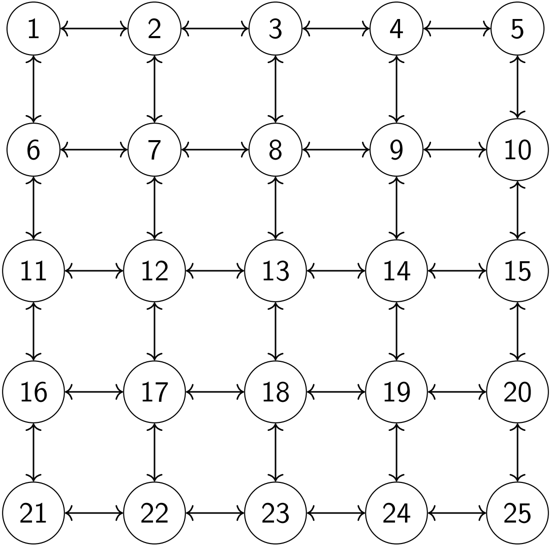
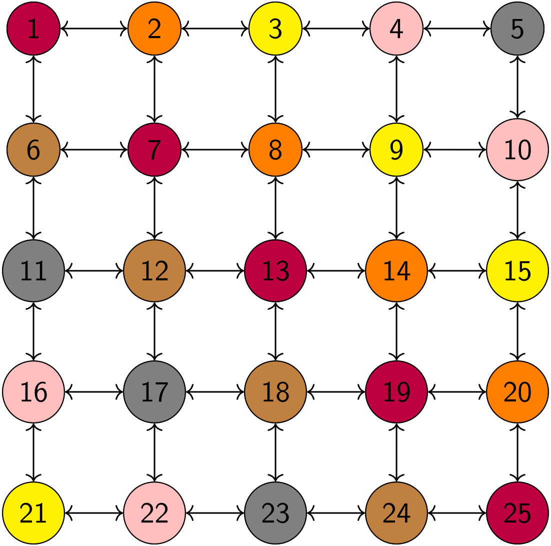
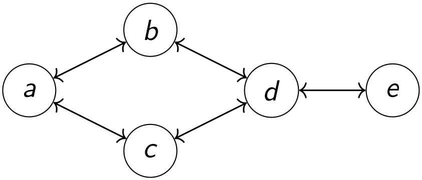


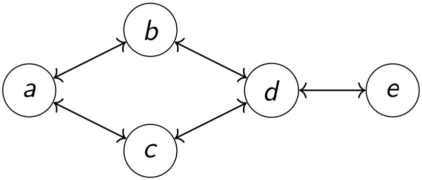
$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$

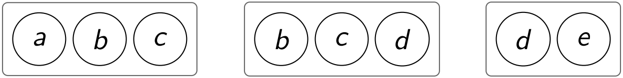


$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$

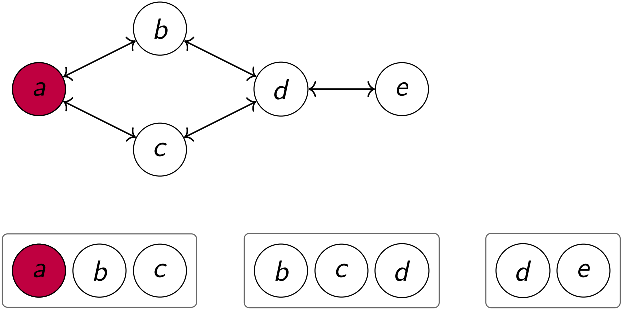


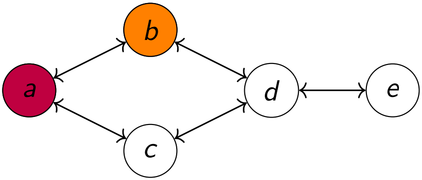
$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$





$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$

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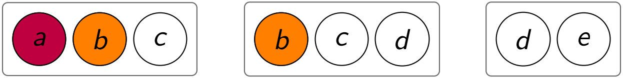
a
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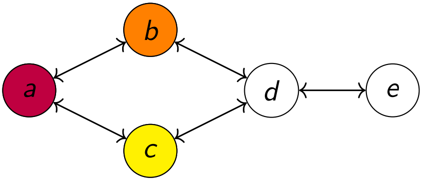
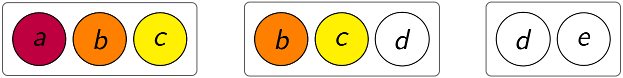
c
b
c
d
d
e

$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$





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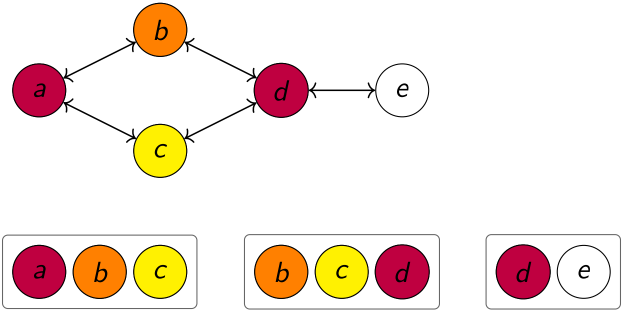

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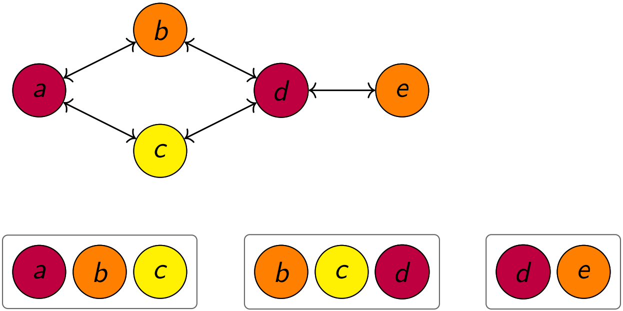
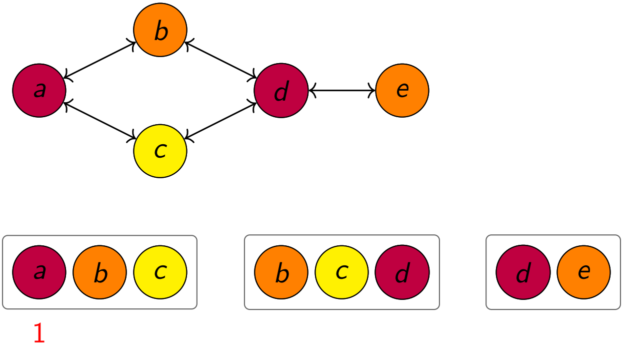
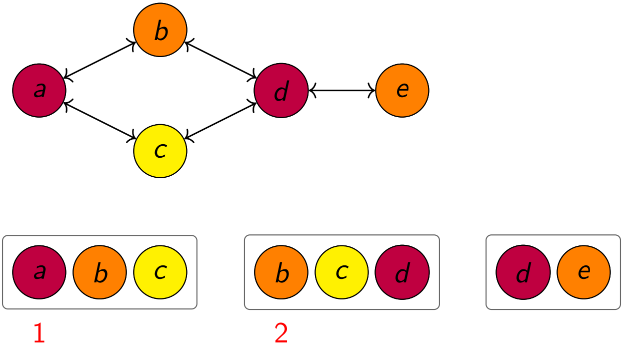


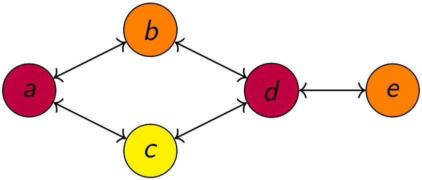
$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$

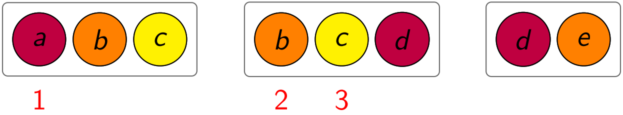


$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$

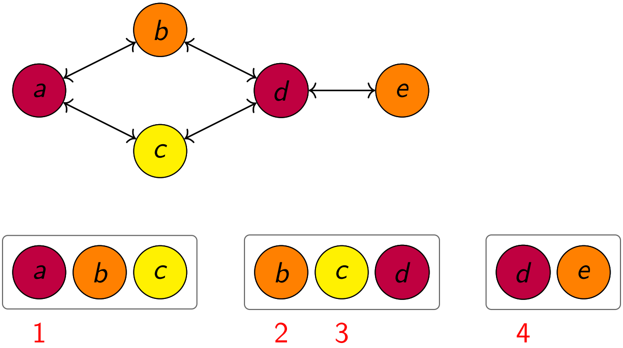


$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$

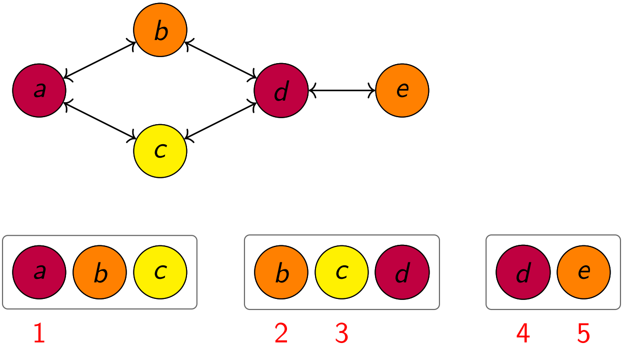




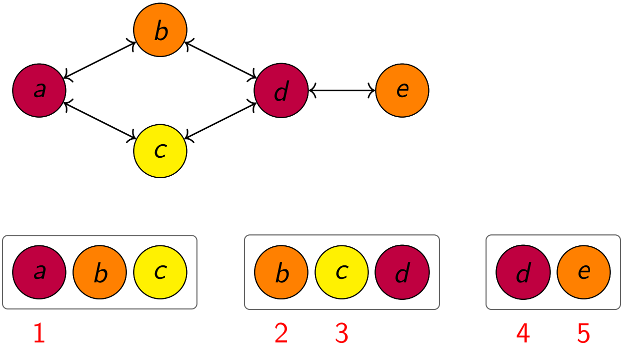
$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$



$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$



$\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$



Theorem. $\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f)) + 1$

Conclusion and perspective

Principal results :

- $\kappa(f, w) = \lceil \log(\chi(G_{f,w})) \rceil$.
- $n/2 \leq \kappa_{q,n} \leq n/2 + \log_q(n)$.
- $n/3 \leq \kappa_{q,n}^{\min}$.
- When $q \geq 4$, $n/2 - \log_q(n) \leq \kappa_{q,n}^{\min}$.
- $\kappa^{\min}(f) \leq \mathrm{Pw}(\mathrm{Ig}(f))$.

Currently studying :

- Smallest n-universal networks.
- Smallest sequential program to compute function.