Quantum Causal Graph Dynamics

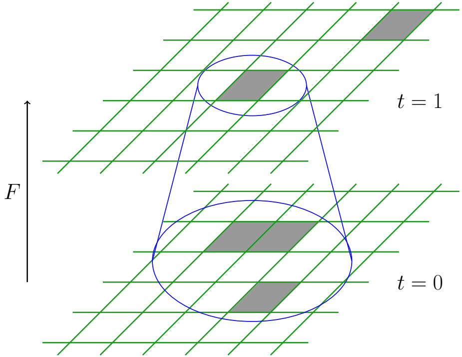
Pablo Arrighi, Simon Martiel

Aix-Marseille Université — CaNa
Atos-Bull R&D — Atos Quantum

July 5, 2017

Digital Physics?

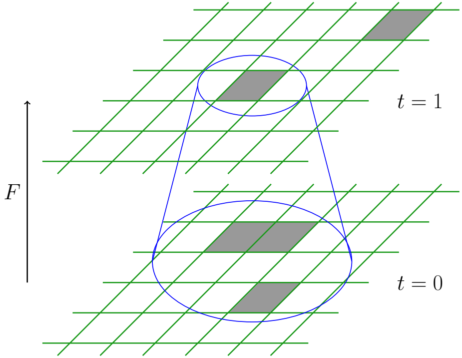
Cellular Automata:



An old CompSci dream : to capture physics in this formalism.

Digital Physics?

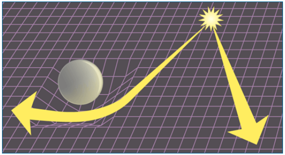
Cellular Automata:



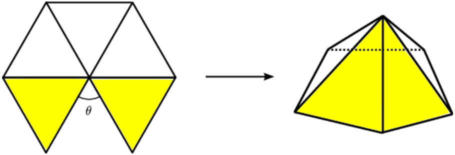
An old CompSci dream : to capture physics in this formalism. $\longrightarrow$ *Theorems about* the extent in which modern physics can be captured in this formalism.

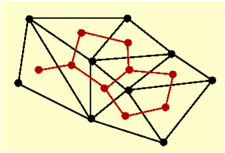
Digital Physics *vs General Relativity*

Digital Physics assumes flat space, yet



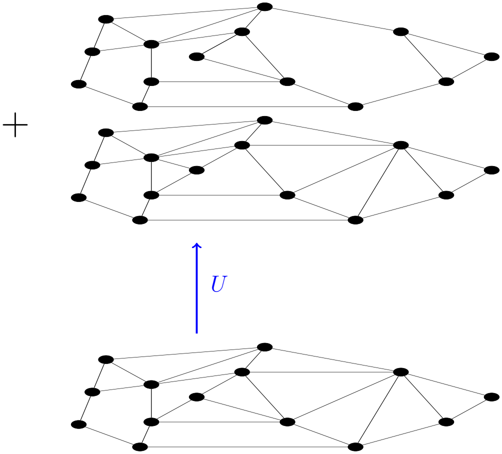
curved. Curved space evolves to extremize curvature





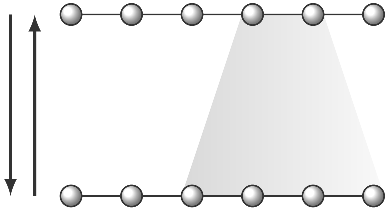
captured as simplicial complex, or graphs.

Digital Physics *vs Quantum mechanics*



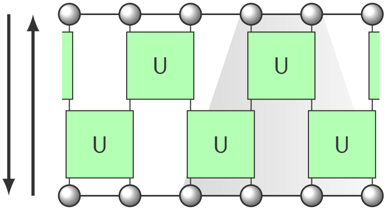
Outline

Causality (Axiomatic principle)



Outline

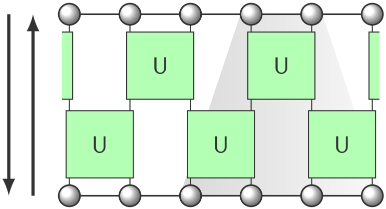
Causality (Axiomatic principle)
⇒ Localizability (Constructiveness)



Outline

Causality (Axiomatic principle)

⇒ Localizability (Constructiveness)

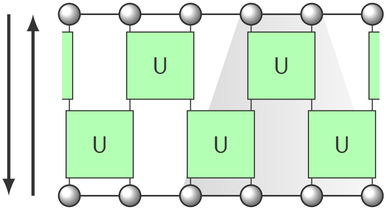


Results:

Reversible CA

Outline

Causality (Axiomatic principle)
⇒ Localizability (Constructiveness)



Results:

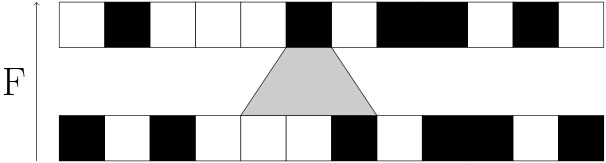
Quantum CA

↑

Reversible CA

Axiomatic principles

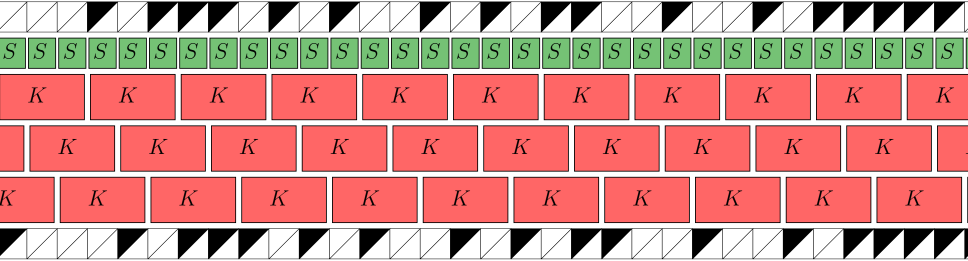
- Causality : $\exists r, \forall u, F(c)^0_u = F(c^r_u)^0_u$.
- Reversibility : $\exists F^{-1}$



Ex: the shift.

Reversible Cellular Automata

Can they always be put in the form of a reversible circuit?

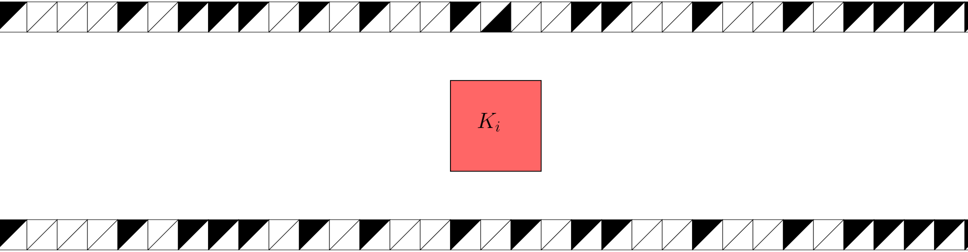


Reversible Cellular Automata

Main layer ◸,◤

Work layer ◺,◢

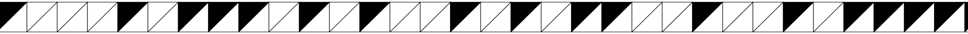
K_i: local reversible gate for updating i:



Main layer ◸,◤

Work layer ◿,◢

K_i: local reversible gate for updating i:

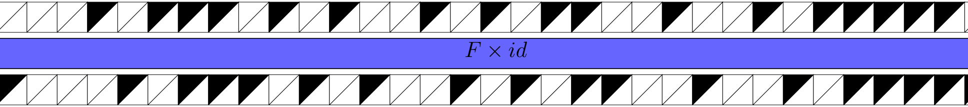


Main layer ◸,◤

Work layer ◿,◢

K_i: local reversible gate for updating i:

- Apply F.

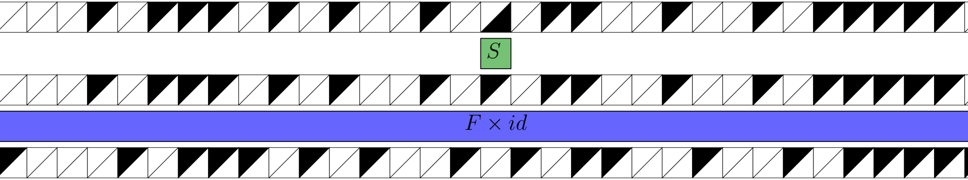


Main layer ◸,◤

Work layer ◿,◢

K_i: local reversible gate for updating i:

- Apply F.
- Swap on cell i.



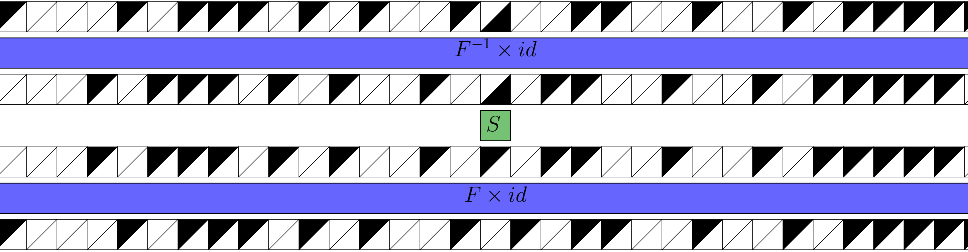
Reversible Cellular Automata

Main layer ◸,◤

Work layer ◿,◢

K_i: local reversible gate for updating i:

- Apply F.
- Swap on cell i.
- Undo F.



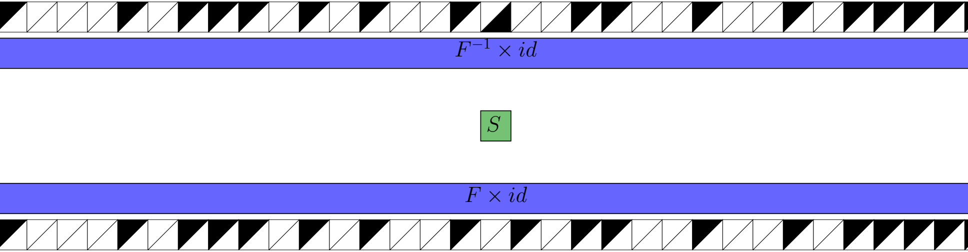
Reversible Cellular Automata

Main layer ◸,◤

Work layer ◺,◢

K_i: local reversible gate for updating i:

- Apply F.
- Swap on cell i.
- Undo F.



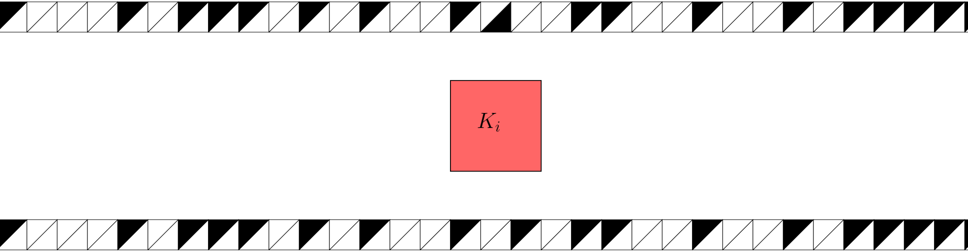
Reversible Cellular Automata

Main layer ◸,◤

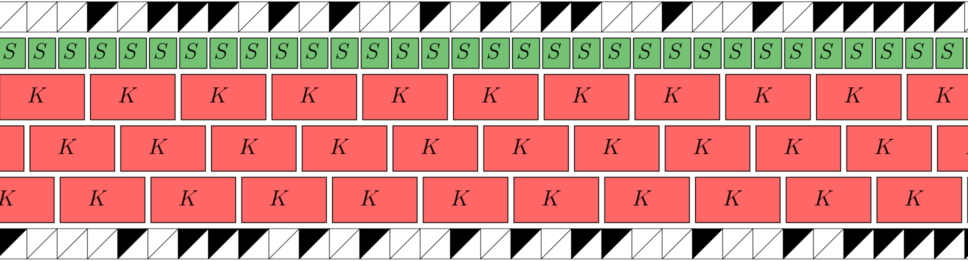
Work layer ◿,◢

K_i: local reversible gate for updating i:

- K_i local, reversible
- K_i and K_j commute



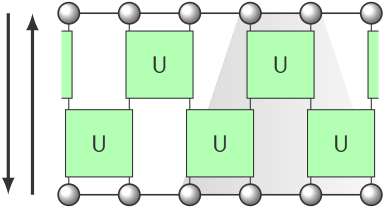
A reversible circuit!

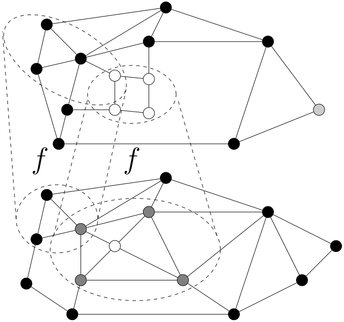


Outline

Causality (Axiomatic principle)

⇒ Localizability (Constructiveness)





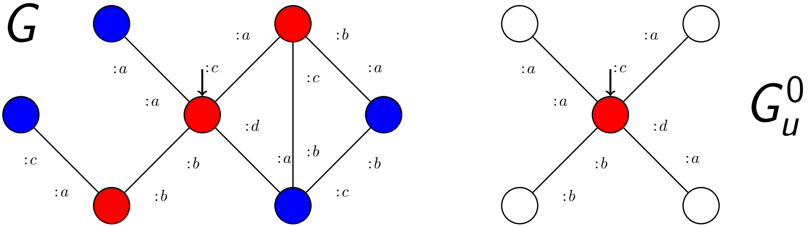
Results:

Quantum CA

↑

Reversible CA → Reversible CGD

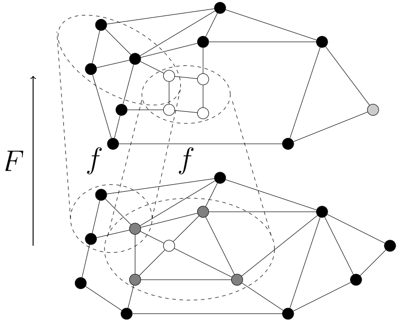
Reversible Causal Graph Dynamics



Reversible Causal Graph Dynamics

Axiomatic principles

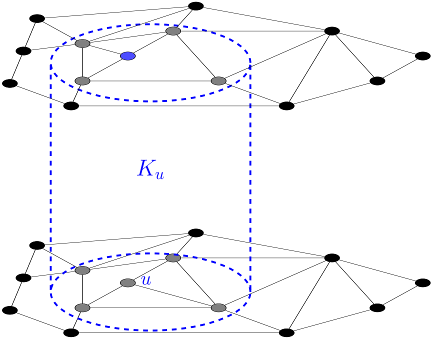
- Causality : $\exists r, \forall u, F(G)^0_u = F(G^r_u)^0_u$. [A.,Dowek]
- Reversibility : $\exists F^{-1}$



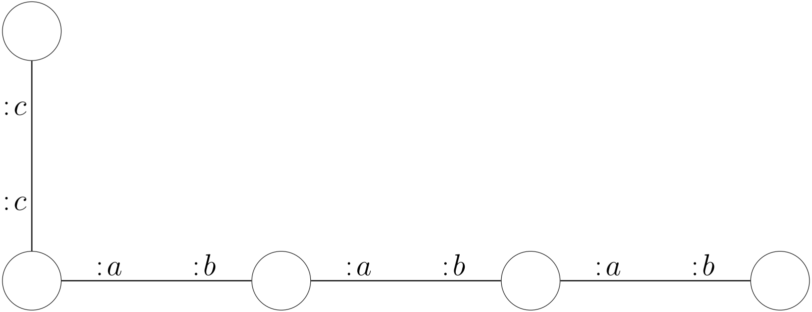
Reversible Causal Graph Dynamics

. . . can always be put in the form of a reversible circuit! [A.,Martiel]

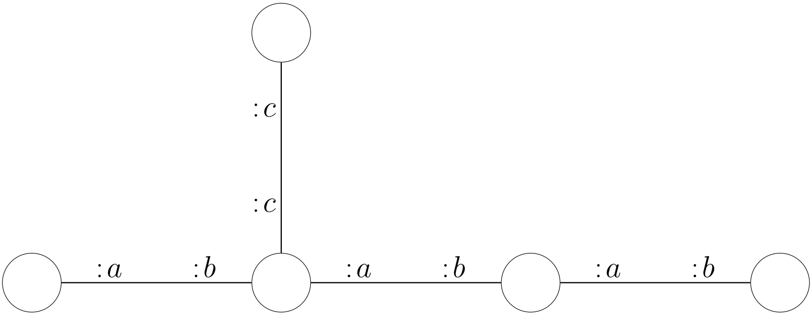
- $F = (\prod \mu_u)(\prod K_u)$
- $K_u = F^{-1} \circ \mu_u \circ F$ is local and reversible.
- K_u and K_v commute,
- μ_u (un)hides vertex u.



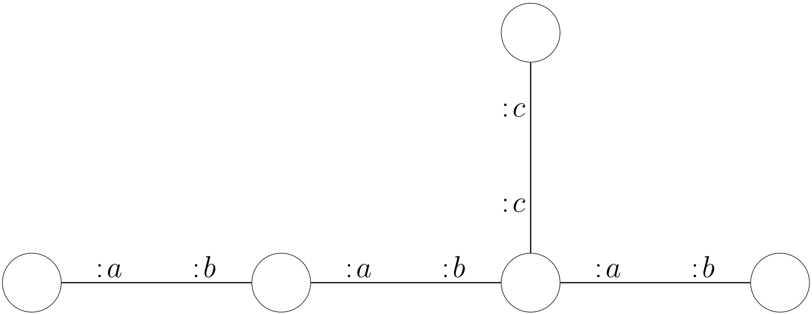
Reversible Causal Graph Dynamics



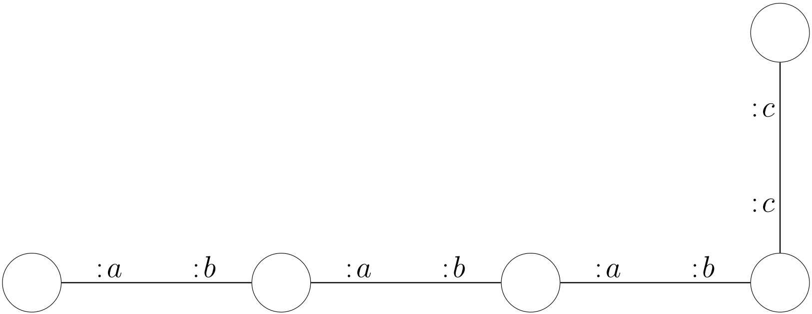
Reversible Causal Graph Dynamics



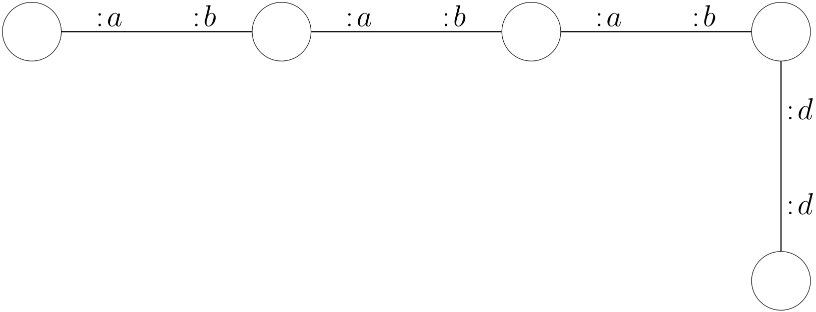
Reversible Causal Graph Dynamics



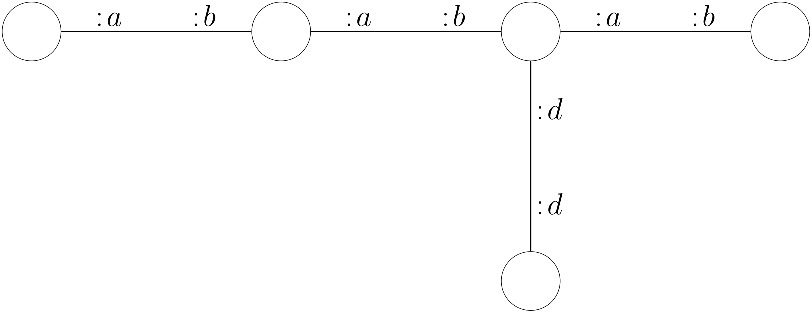
Reversible Causal Graph Dynamics



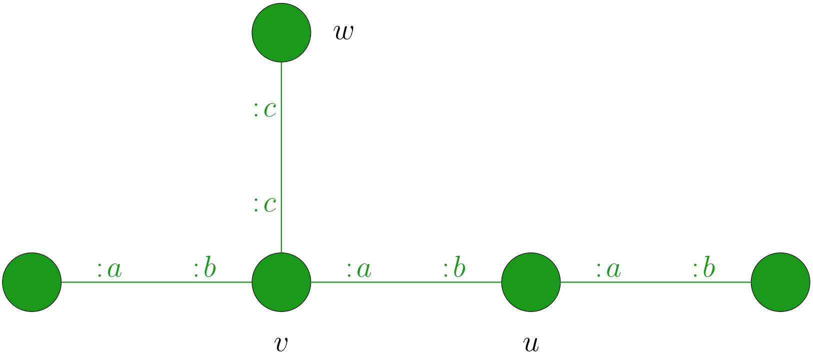
Reversible Causal Graph Dynamics



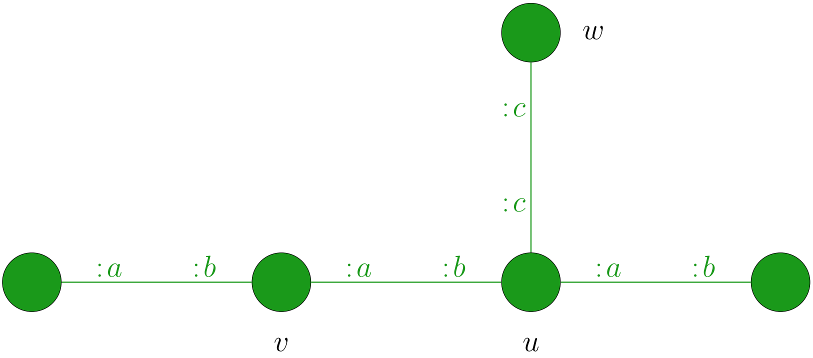
Reversible Causal Graph Dynamics



Reversible Causal Graph Dynamics

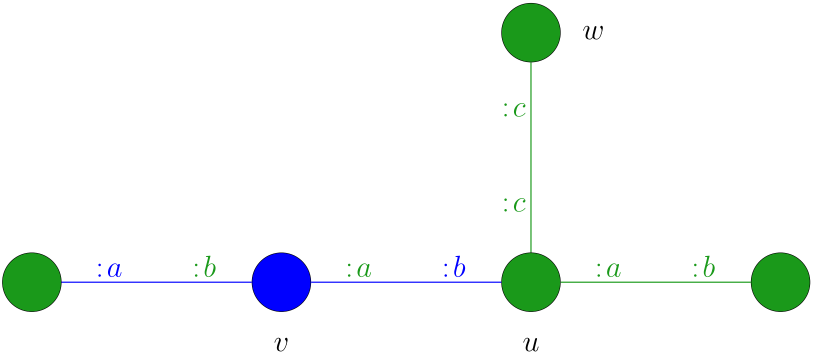


Reversible Causal Graph Dynamics



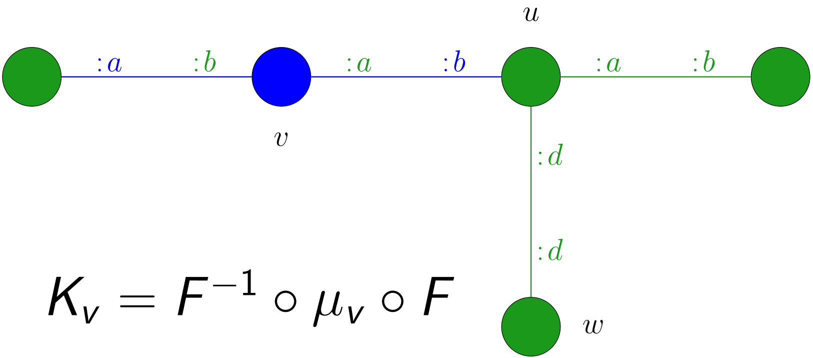
F

Reversible Causal Graph Dynamics



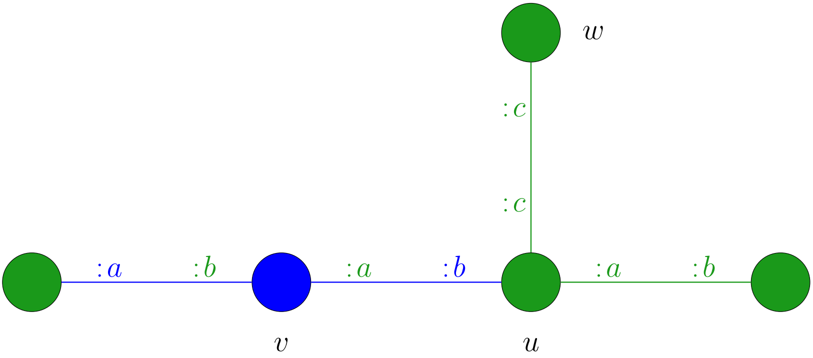
$$\mu_V \circ F$$

Reversible Causal Graph Dynamics



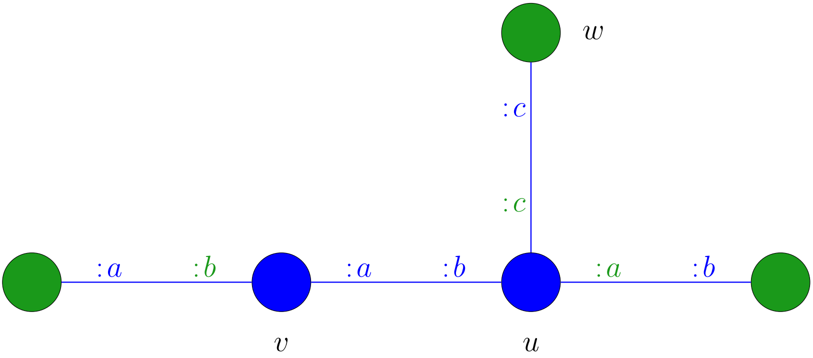
$$K_v = F^{-1} \circ \mu_v \circ F$$

Reversible Causal Graph Dynamics



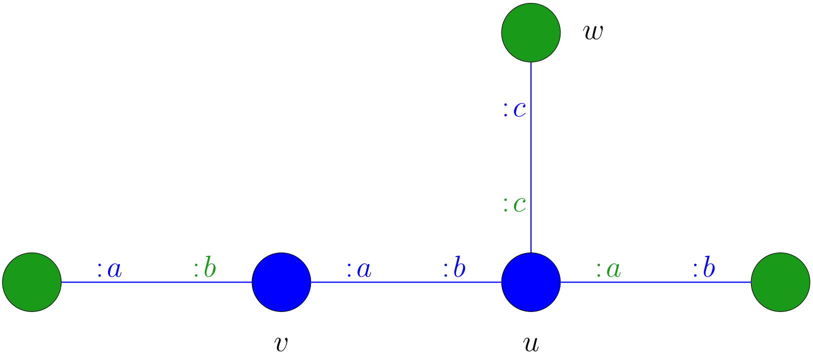
$$F \circ K_v$$

Reversible Causal Graph Dynamics



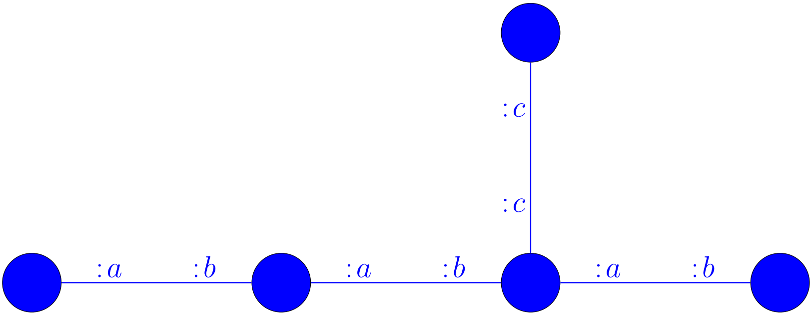
$$\mu_u \circ F \circ K_v$$

Reversible Causal Graph Dynamics



$$K_u \circ K_v$$

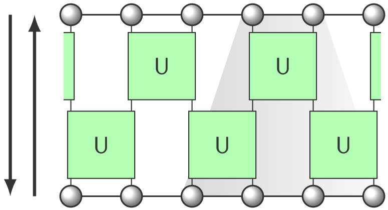
Reversible Causal Graph Dynamics

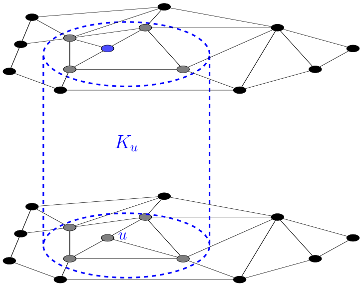


$$\left(\prod K_i\right)$$

Outline

Causality (Axiomatic principle)
⇒ Localizability (Constructiveness)



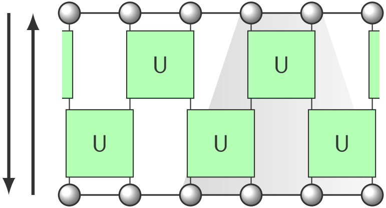


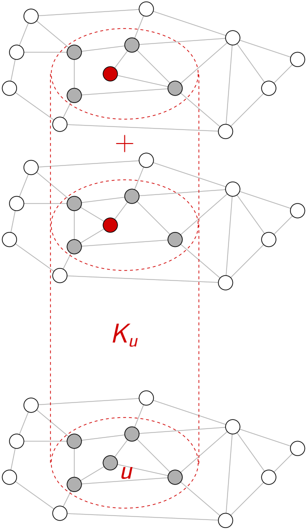
Results:

Quantum CA
↑
Reversible CA → Reversible CGD

Causality? (Axiomatic principle)

⇒ Localizability? (Constructiveness)

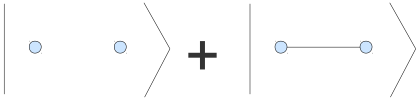




Results:

Quantum CA	→	Quantum CGD
↑		↑
Reversible CA	→	Reversible CGD

Quantum Causal Graph Dynamics



Can these two signal?

Define Causality? Locality?

U causal unitary, A local $\overset{?}{\Rightarrow}$ $UAU^\dagger$ local

Quantum Causal Graph Dynamics

Theorem

Vertex-preserving causal unitary $U \Rightarrow$ [A.,Martiel]

$$U|\psi\rangle = \Big(\prod_{u\in V} \mu_u\Big)\Big(\prod_{u\in V} K_u\Big)|\psi\rangle$$

where the (K_u) *are commuting unitary* r*-local operators.*

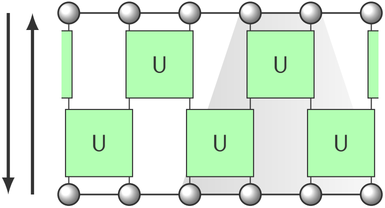
- $K_u = U^\dagger \mu_u U$ is r-local and unitary.
- K_u and K_v commute,
- μ_u (un)hides vertex u.

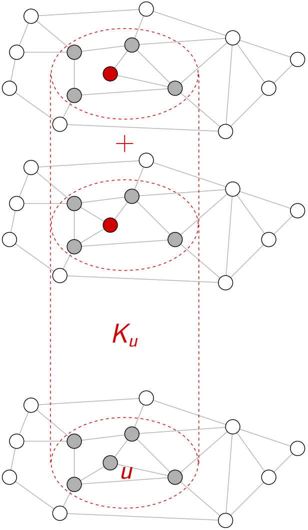
$$\begin{aligned} K_{u_1} K_{u_2} \cdots K_{u_k}|G\rangle &= U^\dagger \mu_{u_1} U U^\dagger \mu_{u_2} U \cdots U^\dagger \mu_{u_k} U|G\rangle \\ &= U^\dagger \mu_{u_1} \mu_{u_2} \cdots \mu_{u_k} U|G\rangle \\ &= U^\dagger \mu_{u_1} \mu_{u_2} \cdots \mu_{u_k} U|G\rangle \\ &= \mu_{u_1} \mu_{u_2} \cdots \mu_{u_k} U|G\rangle \end{aligned}$$

Outline

Causality! (Axiomatic principle)

⇒ Localizability! (Constructiveness)





Results:

Quantum CA → Quantum CGD

↑ ↑

Reversible CA → Reversible CGD

Summary & Perspectives

Updated digital physics evolving superpositions of graphs.

***

Superpositions of graphs are wild objects $\longrightarrow$
Reached a formalization, with the right logical interrelations between concepts of Locality, Causality, Reduced density matrix.
Showed that vertex-preserving causal unitary $\sim$ quantum circuits.

Summary & Perspectives

Updated digital physics evolving superpositions of graphs.

***

Superpositions of graphs are wild objects $\longrightarrow$
Reached a formalization, with the right logical interrelations between concepts of Locality, Causality, Reduced density matrix.
Showed that vertex-preserving causal unitary $\sim$ quantum circuits.

***

Next,

- Lift vertex-preservingness.
- Covariance.
- Study dynamics, emergence.

Annex

Broken Def. (r-Local)

A r-local upon v $\Leftrightarrow$

$$A = A_D \otimes I_{\overline{D}}$$

with $D = (r$-neighbours of $v)$.

Quantum Causal Graph Dynamics

Broken Def. (r-Local)

A r-local upon v $\Leftrightarrow$

$$A = A_D \otimes I_{\overline{D}}$$

with $D = (r$-neighbours of $v)$.

Gen. Def. (r-Local)

A is r-local upon v $\Leftrightarrow \forall G, H$:

$$\langle H|A|G\rangle = \langle H_D|A|G_D\rangle\langle \overline{H}_D|\overline{G}_D\rangle$$

with $D = (r$-neighbours of v in $G) \cup (r$-neighbours of v in $H)$.

- A acts around v, so its only chance of mapping G to H is if they differ around v — whatever that means for them
- Around v, i.e. D, has become a state-dependent notion — since graphs are no longer fixed.

Quantum Causal Graph Dynamics

Broken Lem. (r-Local observable)

A is r-local upon v if and only if, for all ρ:

$$Tr(A\rho) = Tr(A\rho^r_v)$$

Broken Def. (Reduced density matrix)

$$\rho^r_v = \rho_D = Tr_{\overline{D}}(\rho)$$

with $D = (r\text{-neighbours of } v)$.

Quantum Causal Graph Dynamics

Gen. Lem. (r-Local observable)

A is r-local upon v if and only if, for all ρ:

$$Tr(A\rho) = Tr(A\rho_v^r)$$

Broken Def. (Reduced density matrix)

$$\rho_v^r = \rho_D = Tr_{\overline{D}}(\rho)$$

with $D = (r\text{-neighbours of } v)$.

Quantum Causal Graph Dynamics

Gen. Lem. (r-Local observable)

A is r-local upon v if and only if, for all ρ*:*

$$Tr(A\rho) = Tr(A\rho^r_v)$$

Broken Def. (Reduced density matrix)

$$\rho^r_v = \rho_D = Tr_{\overline{D}}(\rho)$$

with $D = $ *(r-neighbours of v).*

Gen. Def. (r-Local)

If $\rho = |G\rangle\langle H|$*, then:*

$$\rho^r_v = |G_D\rangle\langle H_D|\langle\overline{H}_D|\overline{G}_D\rangle$$

with $D = $ *(r-neighbours of v in G)* $\cup$ *(r-neighbours of v in H).*
Then extend by linearity.

Definition (Causal unitary)

U causal unitary $\Leftrightarrow$ $\exists r \,/\, \forall \rho, \forall v$:

$$(U\rho U^\dagger)^0_v = (U\rho^r_v U^\dagger)^0_v.$$

Definition (Causal unitary)

U causal unitary $\Leftrightarrow \exists r \,/\, \forall \rho, \forall v$:

$$(U\rho U^\dagger)^0_v = (U\rho^r_v U^\dagger)^0_v.$$

Things fall into place:

Lemma (Conjugate local is local)

A n-local upon v, and U is causal unitary $\Rightarrow UAU^\dagger$ m-local upon v.